Math 115 Final Exam Wednesday December 17, 2008

1. Consider the surface $x^2 + 2y^2 + 4z^2 = 10$. Find the tangent plane to the surface at $(x,y,z) = (2,1,1)$. Find where this plane intersects the z-axis. This plane intersects the z-axis at z =

   A. 5 B. $2\frac{1}{2}$ C. $1\frac{1}{2}$ D. 0 E. -2 F. $-4\frac{1}{2}$ G. -8 H. -10

2. Find the maximun of $f(x,y) = 3x - 4y$ in the region $x^2 + y^2 \leq 25$. Max =

   A. 4 B. 8 C. 9 D. 15 E. 16 F. 20 G. 25 H. 36

3. The function $f(x,y) = x^3 + y^2 - 3x + 4y$ has exactly one saddle point. The value of f at the saddle point is

   A. 0 B. 1 C. -1 D. 2 E. 5 F. -2 G. 3 H. -3

4. Evaluate $\displaystyle\int_{-2}^{0}\int_{\frac{1}{2}y+1}^{1} e^{-x^2}\,dx\,dy$

   A. $e - 1$ B. $e^2 - 2$ C. $1 - e^{-1}$ D. $2\ln(2)$ E. 0 F. 4 G. $(e-2)/e$ H. $(e-1)/2$

5. There are three red balls and two green balls in a jar. Three balls are drawn out with out replacement. What is the probability there are more read balls than green balls drawn? Probability more red balls drawn =

   A. 4/7 B. 3/5 C. 13/20 D. 7/10 E. 3/4 F. 4/5 G. 6/7 H. 9/10

6. There are three red balls and two green balls in a jar. Three balls are drawn out with replacement, (i.e. after each draw the ball is returned to the jar. What is the probability there are more red balls than green balls drawn? (5^3=125) Probality there are more red balls drawn =

   A. $\frac{63}{125}$ B. $\frac{67}{125}$ C. $\frac{71}{125}$ D. $\frac{77}{125}$ E. $\frac{81}{125}$ F. $\frac{86}{125}$ G. $\frac{91}{125}$ H. $\frac{97}{125}$

7. A fair coin is flipped 4 times. What is the probability that there are more heads produced in the first two flips than in the last two flips. Probability of more heads in first two flips =

   A. 1/8 B. 3/16 C. 1/4 D. 5/16 E. 11/32 F. 3/8 G. 7/16 H. 1/2

8. There are four coins. One has a probability of 1/3 of producing a heads, two are fair and one has a probability of 2/3 of producing a heads. One coin is picked at radom and flipped twice producing a heads and a tails. What is the probability it is one of the fair coins? Prob. of fair coin =

   A. 3/4 B. 9/17 C. 1/2 D. 8/17 E. 4/9 F. 7/17 G. 7/16 H. 3/8

9. The continuous random variable X is distributed over the interval [0,2] with probability density $3x^2/8$. Find the variance of X. Var(X) =

A. 1/10 B. 3/20 C. 1/5 D. 1/4 E. 4/15 F. 3/10 G. 7/20 H. 2/5

10. X and Y are continuously distributed random variables on the square $0 \le x \le 2$ and $0 \le y \le 2$ with a joint distribution function $f(x,y) = x/4$. Compute the probability X > Y given Y < 1. Prob(X > Y|Y < 1) =

A. 1/2 B. 7/12 C. 2/3 D. 3/4 E. 5/6 F. 11/12 G. 12/13

11. The number of clicks of a Geiger is a Poisson process with an average of one click per second. In a given second it is known that there are less then four clicks. What is the expected number of clicks in that second?

A. 1/2 B. 2/3 C. 3/4 D. 4/5 E. 7/8 F. 11/12 G. 15/16 H. 1

12. If R is a 3 x 3 matrix given by $R\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-y\\x\\z\end{bmatrix}$ and if R^{-1} is the inverse matrix for R then

$R^{-1}\begin{bmatrix}1\\2\\3\end{bmatrix}=$ A. $\begin{bmatrix}3\\-2\\1\end{bmatrix}$ B. $\begin{bmatrix}-1\\-2\\3\end{bmatrix}$ C. $\begin{bmatrix}-1\\2\\3\end{bmatrix}$ D. $\begin{bmatrix}2\\3\\1\end{bmatrix}$ E. $\begin{bmatrix}-2\\1\\3\end{bmatrix}$ F. $\begin{bmatrix}2\\-1\\3\end{bmatrix}$ G. $\begin{bmatrix}-1\\2\\3\end{bmatrix}$ H. $\begin{bmatrix}1\\-2\\3\end{bmatrix}$

13. Consider the equations x - 2y = 1 and 2x - Ax = B where A and B are the numbers obtained by rolling two dice numbered 1,2,3,4,5,6. What is the probability these equations have no solution. Prob of no solution =

A. 2/36 B. 4/36 C. 5/36 D. 6/36 E. 7/36 F. 8/36 G. 9/36 H. 11/36

14. There are two independent exponentially distributed random variables A and B where A has a mean of 3 seconds and B a mean of 2 seconds. What is the probability that A occurs more than one second after B? Prob(A > B + 1) =

A. $\frac{3e}{5}^{-1/3}$ B. $\frac{2e}{5}^{-1/3}$ C. $\frac{1e}{3}^{-1/3}$ D. $\frac{3}{5}$ E. $\frac{2}{5}$ F. $\frac{3e}{5}^{-2/3}$ G. $\frac{1}{2}e^{-1}$ H. $1 - e^{-1/2}$

15. For each of the systems of equations below indicate whether there are no solutions (0), a unique solution (1) or infinitely many solutions (∞)

I

$$\begin{aligned} x + y + z &= 1 \\ y + z &= 1 \\ x + 2y + 2z &= 1 \end{aligned}$$

II

$$\begin{aligned} x + y + 2z + w &= 1 \\ y + z + 2w &= 1 \\ z + w &= 4 \\ w &= 3 \end{aligned}$$

III

$$\begin{aligned} x + y + z &= 1 \\ y + z &= 1 \\ x + 2y + 2z &= 2 \end{aligned}$$

Circle one

I: (0 1 ∞) II: (0 1 ∞) III: (0 1 ∞)

16. You are told that $A^2 = AA = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$ and $A^3 = AAA = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix}$.

Find $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and compute the sum of the coefficients $S = a + b + c + d$.

S = A. 2 B. 5 C. 4 D. -1 E. -2 F. 3/2 G. -3/2 H. 1/2

17. Three people A,B and C are playing catch. The probabilities of each throwing to the others are: Prob(A$\to$B) = 1/2 Prob(A$\to$C) = 1/2 Prob(B$\to$A) = 2/3 Prob(B$\to$C) = 1/3 Prob(C$\to$A) = 1/2 Prob(C$\to$B) = 1/2. What is the probability that A will have the ball in the long run?

A. 10/27 B. 1/3 C. 8/27 D. 2/3 E. 1/2 F. 7/9 G. 2/9 H. 5/18

18. It is known that the verbal SAT scores for men and women are normally distributed random variables. For men the mean is 660 and the standard deviation is 30 and for women the mean is 680 and the standard deviation is 20. A college whats to set the mimimun verbal SAT score so men and women have an equal oportunity to be considered. At what level will men and women have an equal oportunity for being considered.

A. 680 B. 690 C. 700 E. 710 F. 720 G. 730 H. Always more women.

19. An unfair coin A has a 4/9 probability of landing heads is tossed 81 times and a fair coin B is tossed 64 times. What is the probability that coin B will produce as many or more heads than A? Circle the closest answer. Indicate what you looked up and how you used it.

A. 95% B. 80% C. 75% D. 50% E. 40% F. 25% G. 15% H. 5%

Answers BGFCDEDBBFGFCA 15.(0,1,∞) CAFF