Name(print)_____  Penn I.D. _____

Signature_____  A correct answer without supporting work will be given little or no credit.

1. The equation for a surface is given by $3x^2 + 2y^2 + z^4 = 6$.  Find the tangent plane to this surface at the point $(x,y,z) = (1,1,1)$ and determine where the plane intersects the z-axis.

    A. 7/2   B. -3   C. 4   D. 3/2   E. 4/3   F. 6   G. 5   H. -2

2. Find the maximum of the function $f(x,y) = x^2 + y$ in the region $x^2 + y^2 \leq 1$.
   Hint. (If the maximum occurs inside the circle the $f_x = f_y = 0$ there.)

Max = A. 0   B. 1/2   C. $1/\sqrt{2}$   D. 1   E. 5/4   F. $\sqrt{2}$   G. 4/3   H. $\sqrt{3}$

3. The function $f(x,y) = x^3 - 3xy + y^3$ has critical points at $(0,0)$ and $(1,1)$.
   Find there types

   A. $\begin{cases} \text{rel min at x=0,y=0} \\ \text{rel min at x=1,y=1} \end{cases}$   B. $\begin{cases} \text{rel min at x=0,y=0} \\ \text{saddle at x=1,y=1} \end{cases}$

   C. $\begin{cases} \text{rel min at x=0,y=0} \\ \text{rel max at x=1,y=1} \end{cases}$   D. $\begin{cases} \text{rel max at x=0,y=0} \\ \text{rel min at x=1,y=1} \end{cases}$   E. $\begin{cases} \text{rel max at x=0,y=0} \\ \text{saddle at x=1,y=1} \end{cases}$

   F. $\begin{cases} \text{saddle at x=0,y=0} \\ \text{rel max at x=1,y=1} \end{cases}$   G. $\begin{cases} \text{saddle at x=0,y=0} \\ \text{rel min at x=1,y=1} \end{cases}$   H. $\begin{cases} \text{saddle at x=0,y=0} \\ \text{saddle at x=1,y=1} \end{cases}$

4. Evaluate $\displaystyle\int_0^2 \int_y^2 e^{\frac{1}{2}x^2}\, dx\, dy$.

    A. 1   B. e   C. 2   D. 1 - e   E. $e^2 - 1$   F. ½(e - 1)   G. $e^{\frac{1}{2}} - 1$   H. e - 2

5. Suppose A, B and C are three mutually independent events and $\Pr(A) = 1/2$ $\Pr(B) = 1/3$ and $\Pr(C) = 1/3$.  What is the probability that exactly one event occurs?.

A. 0   B. $\dfrac{1}{18}$   C. $\dfrac{1}{9}$   D. $\dfrac{1}{6}$   E. $\dfrac{2}{9}$   F. $\dfrac{1}{3}$   G. $\dfrac{4}{9}$   H. $\dfrac{7}{18}$

6. A jar contains 5 red balls and 4 green balls.  If 5 balls are randonly
   drawn out of the jar without replacement.  What is the probability that
   there are more red balls than green balls drawn?

   A. 6/11 B.  9/14 C. 67/126 D. 85/126 E. 2/3  F. 5/9  G. 53/64 H. 11/12

7. There are six pairs of socks (red, blue, gray, white, green and yellow)
   and three are draw out.  What is the probability that there is a pair of
   sock in the three drawn out?

    A. 1/2  B. 1/3  C. 2/11  D. 3/11  E. 1/33  F. 4/33  G. 7/33  H. 11/72


8. Coin A is a fair coin and coin B produces heads 1/3 of the time and
   tails 2/3 of the time.  Each coin is tossed twice.  What is the
   probability that each coin produces the same number of heads.

   A. 5/36  C. 6/36  D. 7/36 E. 9/36 F. 10/36 G. 12/36  H. 13/36

9. A die numbered 1,1,2,2,3,3 (so the probability of producing a 1,2 or 3
   is 1/3) is tossed and the sum of the numbers obtained in each toss is
   computed.  What is the expected number of tosses where the sum will equal
   or exceed three.  ( e.g. 1,1,1 n = 3, 2,2,1 n = 2, 3,2,1 n = 1 etc.)

   A. 1 B. 10/9  C. 4/3  D. 14/9  E. 5/3  F. 16/9  G. 17/9  H. 2


10. There are ten six sided dice.  Four of the dice have all ones, four
    dice have four ones and the remaining sides are blank and the two
    remaining dice have two ones and the remaining sides are blank.  One of
    the ten dice is selected at random and tossed twice.  The die produces
    a one on each toss.  What is the probability it is one of the dice with
    all ones.

   A. 1/3  B. 5/6  C. 11/18  D. 2/3  E. 5/8  F. 9/14  G. 3/4  H. 1


11. The joint probability distribution function for X and Y where $0 \leq X \leq 2$ and
    $0 \leq Y \leq 1$ is f(x,y) = xy.  Compute the probability that X > Y.

    A. 0  B. 2/9 C. 1/3  D. 4/9  E. 1/2  F. 5/9  G. 7/8  H. 15/16
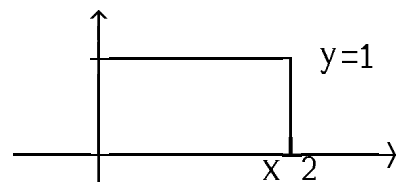
12. What is the variance of a random variable distributed on the interval [0,1] with probability density function $f(x) = 2 - 2x$ for $0 \le x \le 1$? Variance =
A. 1/22   B. 1/18   C. 1/9   D. 2/9   E. 5/18   F. 1/3   G. 1/6   H. 1/2


13. Suppose X is a exponentially distributed random variable with mean two seconds and Y is a exponentially distributed random variable with mean three seconds.  (The probability density function for an exponentially distributed randon variable with mean m is $f(t) = (1/m)e^{-t/m}$ for $t \ge 0$) Given that the random variables X and Y are independent compute the probability that neither X or Y occurs in the first two seconds.  Prob =

A. 1/3   B. 2/3   C. 1/2   D. $e^{-2}$   E. $e^{-5/3}$   F. $1-e^{-1}$   G. $e^{-1}$   H. e/4


14. The number of clicks of a Geiger counter is a Poisson process.  The Geiger counter clicks 6 times a minute on the average.   In a thirty second interval it is known that the Geiger counter clicks at least twice.  (e.g. $n \ge 2$).  What is the probability it clicks at most three times.  $Pr(n \le 3| n \ge 2) =$

A. 1/2 B. 2/3 C. $9e^{-3}$ D. $4e^{-2}$ E. $\dfrac{(9/2)\ e^{-3}}{1 - 4e^{-3}}$ F. $\dfrac{9e^{-3}}{1 - 4e^{-3}}$ G. $\dfrac{2e^{-2}}{1 - e}$ H. $11e^{-3}$

15. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.  Find the sum of its entries.
    If $A^{-1} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$ the the sum of all the $c_{ij}$ is =

   A. –1 B. –½ C. 0 D. ½ E. 1 F. 3/2 G. 2 H. 5/2


16. For what values of k if any does the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 3 & k \end{bmatrix}$ have an inverse?

A. all values of k.   B. No values of k.   C. Only for k = 6.   D. Only for k ≠ 6.

E. Only for k = 1.   F. Only for k ≠ 1.   G. Only for k = 2.   H. Only for k ≠ 6.

17. Peter, Paul and their younger sister Mary are playing catch.  Peter and
    Paul throw to Mary 1/4 of the time and to each other 3/4 of the time.
    Mary throws to Paul and Peter with equal probability 1/2.  What is the
    average probablility that Mary will have the ball in the long run?

    A. 1/5  B. 1/6  C. 2/7  D. 3/7  E. 3/13  F. 2/11  G. 3/11  H. 4/11


18. The grade distribution in a large class is a normal distribution with
    mean 75 and standard deviation 10.  If twenty percent of the class get
    A's what is the lowest grade that will produce and A?  Circle the
    closest answer.
    A. 76  B. 78  C. 80  D. 82  E. 84  F. 86  G. 88  H. 90


19. Suppose there are coins.  Coin A is unfair producing heads 3/5 of the time
    and a tails 2/5 of a time while coin B is fair and produces heads produces
    heads and tails with equal probability.  Each coin is flipped 100 times.
    What is the probablity that coin B produces as many or more heads than coin
    A?  You may use either the table of the standard normal distribution below
    or the one on the last page of the exam.  Again you must show what you
    looked up and how you used it.  Circle the closest answer.

1/3=0.33 2/3=0.67 1/6=0.17 5/6=0.83 1/7=0.14, 2/7=0.29 3/7=0.43 4/7=0.57
5/7=0.71 6/7=0.86 1/8=0.13 3/8=0.38 5/8=0.63 7/8=0.88 n/9=0.nnnn... n=1,2,..,8

    A. 20%  B. 13%  C. 8%  D. 4%  E. 2%  F. 0.6%  G. 0.2%  H. 0.03%


Answers AEGEGBBHFDGBEFFDAEC
         B