FINAL EXAM

Math 115, Probability and Matrices
Spring 2007, University of Pennsylvania

Name: _______________
Professor: Drumm or Temkin (circle one)
Recitation: _________________

One sheet of notes is permitted but no calculators are allowed. Circle the letters for your answer for each problem, the front page is for the graders. Each problem is worth 10 pts. Work is required for full credit. You may be asked for your Penn ID. The time limit is 2 hours.

1			10	
2			11	
3			12	
4			13	
5			14	
6			15	
7			16	
8			17	
9			18	

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1. Find the z-coordinate of the point at which the plane tangent to the surface $z = x\sqrt{y}$ at the point $(3,4,6)$ intersects the z-axis.

a. -3

b. -2

c. -1

d. 0

e. 1

f. 2

g. 3

h. 4

2. A function $h(x,y)$ satisfies the following conditions: $h_x(0,0)=2$ and $h_y(0,0)=3$. Given that $f(t)=h\left(e^t-1,\cos(t)-1\right)$, find the value of $f'(0)=\left.\frac{df}{dt}\right|_{t=0}$.

a. -3

b. -2

c. -1

d. 0

e. 1

f. 2

g. 4

h. 5

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3. Evaluate $\iint_R e^{\sqrt{x}}\,dxdy$, where R is the region bounded by $x=1$, $x=4$, $y=0$ and $y=\dfrac{1}{\sqrt{x}}$.

a. $2\sqrt{e}$

b. $\frac{1}{2}\left(\sqrt{e}-e\right)$

c. $2\sqrt{e}$

d. $\frac{1}{2}\left(e^2-e\right)$

e. $\frac{1}{2}\left(\sqrt{e}-e\right)$

f. e^2-e

g. $2\left(e^2-e\right)$

h. $2e^2$

4. A watch manufacturer can produce $P(x,y)=50x^{0.4}y^{0.6}$ watches, where x is the units of labor and y is the units of capital. Assuming a maximum exists, how many units of labor and capital should be used, i.e. what (x,y), should be used to maximize the number of watches, with the constraint that there is a budget of $20,000 where each unit of labor costs $100 and each unit of capital costs $200, i.e. the constraint is $100x+200y=20,000$.

a. $(80,60)$

b. $(80,80)$

c. $(60,80)$

d. $(80,40)$

e. $(40,40)$

f. $(40,80)$

g. $(0,100)$

h. $(200,0)$

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5. For the function $f(x,y)=(x^2+y^2)e^{-x}$, find the one true statement about the critical points.

   a. There are no critical points
   b. A local max occurs at $(0,2)$
   c. A local min occurs at $(0,2)$.
   d. A saddle occurs at $(0,2)$
   e. A local max occurs at $(0,0)$.
   f. A saddle occurs at $(0,0)$.
   g. A local min occurs at $(2,0)$.
   h. A saddle occurs at $(2,0)$.

6. A committee consists of 4 men and 6 women. A subcommittee of 3 members is chosen at random. What is the probability that the number of women is larger than the number of men in the subcommittee?

a. 1/8

b. 1/4

c. 3/8

d. 1/3

e. 1/2

f. 5/8

g. 2/3

h. 3/4

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7. In an urn there are 5 pink balls and 3 aqua balls. Three balls are chosen at one time and it is known that there are more pink than aqua balls chosen. What is the probability that no aqua balls are chosen?

a. 5/28	e. 2/5
b. 3/14	f. 1/2
c. 1/4	g. 5/8
d. 1/3	h. 3/5

8. There are 6 coins in a wallet. One coin is weighted so that the probability of heads is $1/4$. Two coins are fair, i.e. the probability of heads is $1/2$. Finally, three coins are weighted so that the probability of heads is $3/4$. At coin is drawn at random and tossed. Given that the result is tails, what is the probability that a fair coin was chosen?

   a. $1/5$
   b. $1/4$
   c. $1/3$
   d. $2/5$
   e. $1/2$
   f. $3/5$
   g. $2/3$
   h. $3/4$

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9. A random variable X has a p.d.f. of $f(x)=\begin{cases} c-2x & \text{for } 0\le x\le 1 \\ 0 & \text{otherwise} \end{cases}$. Compute the variance of X, i.e. $\text{Var}(X)$. (Hint: you will have to find c first.)

a. $1/18$

b. $1/24$

c. $1/9$

d. $5/24$

e. $1/20$

f. $1/3$

g. $1/2$

h. 1

10. In a lottery, 6 “winning” numbers are drawn from the numbers 1-36. Each completed lottery ticket contains 6 different numbers between 1 and 36. What is the expected number of “winning” numbers on any one ticket?

a. $1/36$

b. $1/6$

c. $1/3$

d. $1/2$

e. $5/6$

f. 1

g. 2

h. 6

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11. For random variables X and Y with joint probability density function

$$f(x,y)=\begin{cases} 2xe^{-y} & \text{for } 0<x<1,\ y>0 \\ 0 & \text{otherwise} \end{cases}$$

find the probability that $Y > X$.

a. $1-(1/e)$

b. $2-(4/e)$

c. $1-(2/e^2)$

d. $e-2$

e. $3-(2/e)$

f. $2-(3/e)$

g. $2/e$

h. $4/e$

12. A large number of exams are approximately normally distributed with mean $\mu = 80$ and standard deviation $\sigma = 10$. Twenty-five exams are chosen at random. Using the very short table below, which of the following is closest to the probability that the average of these exams is greater than 83?

a. 0.01

b. 0.07

c. 0.38

d. 0.62

e. 0.93

f. 0.99

Values in the table are approximations of $\phi(z) = \Pr(Z < z)$

z	$\phi(z)$		z	$\phi(z)$		z	$\phi(z)$
0.0	0.5000		**1.0**	0.8413		**2.0**	0.9772
0.1	0.5398		**1.1**	0.8643		**2.1**	0.9821
0.2	0.5793		**1.2**	0.8849		**2.2**	0.9861
0.3	0.6179		**1.3**	0.9032		**2.3**	0.9893
0.4	0.6554		**1.4**	0.9192		**2.4**	0.9918
0.5	0.6915		**1.5**	0.9332		**2.5**	0.9938
0.6	0.7257		**1.6**	0.9452		**2.6**	0.9953
0.7	0.7580		**1.7**	0.9554		**2.7**	0.9965
0.8	0.7881		**1.8**	0.9641		**2.8**	0.9974
0.9	0.8159		**1.9**	0.9713		**2.9**	0.9981
1.0	0.8413		**2.0**	0.9772		**3.0**	0.9987

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13. A recently discovered elemement Su (from Super-Uranium) is so radioactive that its atom lives on average $1/1,000^{\text{th}}$ of a second. The scientists who discovered the first atom of Su were very lucky because the atom lived more than $1/200^{\text{th}}$ of a second. Given that the atom lived more than $1/200^{\text{th}}$, what is the probability that the atom lived more than $1/100^{\text{th}}$ of a second? (Hint: use that the duration of an atom's life is distributed exponentially, i.e. its p.d.f. is $\beta e^{-\beta t}$.

a. $\dfrac{e^{-5}}{1-e^{-5}}$

b. $\dfrac{e^{-5}-e^{-10}}{1-e^{-5}}$

c. $\dfrac{1}{1-e^{-5}}$

d. $1-e^{-5}$

e. e^{-5}

f. $\dfrac{e^{-10}}{1-e^{-10}}$

g. $e^{-5}-e^{-10}$

h. $1-e^{-10}$

14. To pass a level in the computer game "Survival" a player must go 5 minutes without making a fatal mistake. Assuming

    a. $1/36$

    b. $1/6$

    c. $1/3$

    d. $1/2$

    e. 1

    f. 2

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15. In a lottery, 6 “winning” numbers are drawn from the numbers 1-36. Each completed lottery ticket contains 6 different numbers between 1 and 36. What is the expected number of “winning” numbers on any one ticket?

    a. $1/36$

    b. $1/6$

    c. $1/3$

    d. $1/2$

    e. 1

    f. 2

16. In a lottery, 6 “winning” numbers are drawn from the numbers 1-36. Each completed lottery ticket contains 6 different numbers between 1 and 36. What is the expected number of “winning” numbers on any one ticket?

a. $1/36$

b. $1/6$

c. $1/3$

d. $1/2$

e. 1

f. 2

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17. In a lottery, 6 "winning" numbers are drawn from the numbers 1-36. Each completed lottery ticket contains 6 different numbers between 1 and 36. What is the expected number of "winning" numbers on any one ticket?

   a. $1/36$

   b. $1/6$

   c. $1/3$

   d. $1/2$

   e. 1

   f. 2

18. In a lottery, 6 “winning” numbers are drawn from the numbers 1-36. Each completed lottery ticket contains 6 different numbers between 1 and 36. What is the expected number of “winning” numbers on any one ticket?

   a. $1/36$

   b. $1/6$

   c. $1/3$

   d. $1/2$

   e. 1

   f. 2