Math 114 Spring 2011 Final Exam

1. The set of points equidistant from the points (2, -1, 1) and (4, 3, -5) is a plane. What is the equation of this plane?

(A) $3x + y - 2z = 0$	(B) $2x + 4y - 6z = -6$
(C) $x + 2y - 3z = 11$	(D) $2x + 14y + 10z = 15$
(E) $6x + 2y - 4z = 5$	(F) $x + y + z = 2$
(G) $2x + 2y + 2z = 7$	(H) $x + 7y + 5z = 0$

2. In Citizens Bank Park, where the Philadelphia Phillies play baseball, the right-field fence is 330 feet from home plate (where the batter stands when he hits), and the fence is about 13 feet high. First-baseman Ryan Howard hits a home run over the right-field fence that starts out 3 feet above home plate with horizontal velocity 66 ft/sec toward the wall and initial upward velocity 84 ft/sec. By how many feet does the ball clear the top of the fence? (Assume the acceleration due to gravity is 32 ft/sec² and ignore wind resistance.)

(A) 2 feet	(B) 4 feet	(C) 6 feet	(D) 8 feet
(E) 10 feet	(F) 12 feet	(G) 14 feet	(H) 16 feet

3. Let P be the plane tangent to the graph of $x^2 e^{yz} = 4$ at the point (2, 0, 3). What point on P is closest to the origin?

(A) $(-1, 1, 3)$	(B) $(-1, 1, 0)$	(C) (2, 0, 3)	(D) $(\frac{1}{2}, 0, \frac{3}{2})$
(E) $(\frac{1}{5}, 0, \frac{3}{5})$	(F) $(\frac{3}{5}, \frac{1}{5}, 0)$	(G) $(\frac{1}{5}, \frac{3}{5}, 0)$	(H) $(\frac{2}{5}, 0, \frac{3}{5})$

4. Calculate the arclength of the curve given parametrically by

$$x = 2t^2, \qquad y = \sqrt{3}t^4, \qquad z = t^6$$

for $0 \leq t \leq 2$.

- (A) 8 (B) $24\sqrt{3}$ (C) 36 (D) $64\sqrt{3}$
- (E) 72 (F) $81\sqrt{3}$ (G) 108 (H) $144\sqrt{3}$

5. The ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

bounds a region of volume $V = \frac{4}{3}\pi abc$. At a moment when a = 1, b = 3, and c = 5, a is changing at a rate of +2 and b is changing at a rate of -3, at what rate must c be changing for the volume to remain constant?

(A)
$$-7$$
(B) -5 (C) -3 (D) -1 (E) 1(F) 3(G) 5(H) 7

6. The function $f(x, y) = x^3 - 6xy - 3y^2$ has

- (A) one local maximum and one saddle point
- (B) one local minimum and one saddle point
- (C) one local maximum and one local minimum
- (D) two local maxima
- (E) two local minima
- (F) two saddle points
- (G) a local maximum and no other critical points
- (H) a saddle point and no other critical points

7. Find the maximum value of the function f(x, y, z) = xy + yz on the surface of the sphere $x^2 + y^2 + z^2 = 4$.

(A) 0	(B) $\frac{1}{\sqrt{2}}$	(C) $\frac{1}{2\sqrt{2}}$	(D) $\frac{1}{2}$
(E) 1	(F) $\sqrt{2}$	(G) 2	(H) $2\sqrt{2}$

8. Calculate $\iint_T x^2 dA$ where T is the triangular region with vertices (0,0), (1,0) and (1,2).

(A) 1/3	(B) 1/2	(C) $2/3$	(D) 1
(E) $4/3$	(F) 3/2	(G) $5/3$	(H) 2

9 . Calculate \int	$\int_{0}^{2} \int_{x^2}^{4} \frac{e^{\sqrt{y}}}{y} dy dx$		
(A) e^{2}	(B) $2e^2$	(C) $2e^2 - 1$	(D) $2e^2 - 2$
(E) $e^{\sqrt{2}}$	(F) $2e^{\sqrt{2}}$	(G) $2e^{\sqrt{2}} - 1$	(H) $2e^{\sqrt{2}} - 2$

10. Find the volume of the solid bounded above by the paraboloid $z = 5 - x^2 - y^2$ and below by the paraboloid $z = 4x^2 + 4y^2$.

(A) $\frac{\pi}{4}$	(B) 1	(C) $\frac{\pi}{2}$	(D) π
(E) $\frac{3\pi}{2}$	(F) 2π	(G) $\frac{5\pi}{2}$	(H) $\frac{7\pi}{2}$

11. Use the transformation u = 3x + y, v = x + 2y to evaluate

$$\iint_R (3x^2 + 7xy + 2y^2) \, dA$$

where R is the region in the plane bounded by the lines 3x + y = 1, 3x + y = 2, x + 2y = 0and x + 2y = 1.

(A)
$$\frac{3}{20}$$
(B) $\frac{3}{16}$ (C) $\frac{1}{8}$ (D) $\frac{1}{10}$ (E) $\frac{1}{2}$ (F) $\frac{3}{4}$ (G) $\frac{1}{4}$ (H) $\frac{5}{4}$

12. Compute the volume of the solid bounded by the four surfaces x + z = -1, x + z = 1, $z = 1 - y^2$, and $z = y^2 - 1$.

(A) $\frac{2}{3}$	(B) $\frac{5}{3}$	(C) $\frac{8}{3}$	(D) $\frac{11}{3}$
(E) 4	(F) $\frac{16}{3}$	(G) $\frac{20}{3}$	(H) $\frac{22}{3}$

13. Compute the work done by the force field

$$\vec{F} = (6xy - y^3)\mathbf{i} + (3y^2 + 3x^2 - 3xy^2)\mathbf{j}$$

along the path $\mathbf{c}(t) = (\cos(t), \sin(t))$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$.

(E)
$$-\frac{1}{2}$$
 (F) $-\frac{1}{4}$ (G) -2 (H) 2

14. Evaluate

$$\int_C xy^3 \, dx + 3x^2 y^2 \, dy$$

where C is the boundary of the region in the first quadrant enclosed by the x-axis, the line x = 1 and the curve $y = x^3$, traversed counter-clockwise.

(A) 0(B)
$$\frac{1}{2}$$
(C) $\frac{1}{3}$ (D) $\frac{1}{4}$ (E) $\frac{1}{6}$ (F) $\frac{1}{11}$ (G) 1(H) 2

15. Let y(t) be the solution to the differential equation

$$t\frac{dy}{dt} = t^2 + 2y$$

satisfying y(1) = 2. What is y(2)?

(A)
$$2 + 4 \ln 2$$
 (B) $8 + 4 \ln 2$ (C) $4 + 2 \ln 2$ (D) $8 + 8 \ln 2$

(E)
$$2 + 8 \ln 2$$
 (F) $4 + 4 \ln 2$ (G) $2 + 2 \ln 2$ (H) $4 + 8 \ln 2$