Math 104 Final Exam
April 27, 2006

Name _____

Penn Student ID _____

Lecturer (circle the name of exactly 1 lecture and exactly 1 TA):

Lecturer:	TA's:	TA's:
Rob Schneiderman	Colin Diemer	Chenxu He
Erik van Erp	Tong Zhu	Chris Jankowski
Mark Daniel Ward	David Favero	Tian Liang

1. _____	6. _____	11. _____	16. _____
2. _____	7. _____	12. _____	17. _____
3. _____	8. _____	13. _____	18. free response
4. _____	9. _____	14. _____	19. free response
5. _____	10. _____	15. _____	20. free response

Score: _____ (100 points possible)

1. The testing booklet contains 20 pages of questions.
2. No calculators are permitted.
3. One piece of paper (8.5 in. by 11 in.) is permitted, with writing on both sides allowed.
4. Make sure to supply answers to all the questions. There is no penalty for guessing.
5. No credit will be given for problems with no answers or problems with multiple answers.
6. No partial credit will be given on Problems 1–17.
7. Write all calculations on the pages provided. Extra pages are available if needed.

1. Find the volume of the solid obtained by rotating the region bounded by the curves

$$y = x^2, \qquad y = 0, \qquad x = 2$$

about the line $x = 4$.

A.) $10\pi/3$ B.) $16\pi/3$ C.) $20\pi/3$ D.) $32\pi/3$ E.) $40\pi/3$ F.) $64\pi/3$

2. A tank with a square base and rectangular sides is filled with a liquid of density $3\,\text{lb}/\text{ft}^3$. The tank is exactly 2 feet long, 2 feet wide, and 5 feet tall. Find the work required to pump the liquid out of the top of the tank.

A.) 8 ft-lb B.) 20 ft-lb C.) 32 ft-lb D.) 40 ft-lb E.) 150 ft-lb F.) 300 ft-lb

3. Find the area of the region bounded by the curves

$$y = x^2 - 2x + 1 \qquad \text{and} \qquad y = -x^2 + 4x + 1\,.$$

A.) 1 B.) 3 C.) 6 D.) 9 E.) 12 F.) 24

4. Find the correct value of the limit

$$\lim_{x \to 0} \frac{(\cos x) - 1}{x^2} \, .$$

A.) 0 B.) 1 C.) -1 D.) $-1/2$ E.) $\pi/2$ F.) ∞

5. Evaluate the integral

$$\int_2^5 \frac{1}{4-x}\,dx\,.$$

A.) 0 B.) $\ln 2$ C.) $-\ln 2$ D.) 1 E.) 3 F.) the integral diverges

6. Evaluate

$$\int_0^1 x \ln x \, dx \, .$$

A.) -1 B.) $-1/2$ C.) $-1/4$ D.) 0 E.) 1 F.) the integral diverges

7. Evaluate

$$\int_0^{\pi/2} \cos^3 x \, dx \, .$$

A.) 0 B.) 1/3 C.) 1/2 D.) 2/3 E.) 1 F.) the integral diverges

8. Evaluate

$$\int_0^1 (2x+1)e^{x^2+x}\,dx\,.$$

A.) $3e^2$ B.) $e-1$ C.) $3e^2-1$ D.) $2e^2$ E.) e^2-1 F.) $\frac{1}{3}e^2-1$

9. Evaluate

$$\int_3^4 \frac{1}{(x-1)(x-2)}\,dx\,.$$

A.) $\ln(1/2)$ B.) $\ln(2/3)$ C.) $\ln(3/4)$ D.) $\ln(4/3)$ E.) $\ln(3/2)$ F.) $\ln(2)$

10. Evaluate

$$\int_{-1}^{1} \sqrt{1-x^2}\, dx\,.$$

A.) 0 B.) $\pi/8$ C.) $\pi/4$ D.) $\pi/2$ E.) π F.) 2π

11. Set up, but do not evaluate, an integral for the length of the curve $y = \ln(\cos x)$ over the interval $0 \leq x \leq \pi/6$.

A.) $\int_0^{\pi/6} \sin x \, dx$

B.) $\int_0^{\pi/6} \cos x \, dx$

C.) $\int_0^{\pi/6} \tan x \, dx$

D.) $\int_0^{\pi/6} \csc x \, dx$

E.) $\int_0^{\pi/6} \sec x \, dx$

F.) $\int_0^{\pi/6} \cot x \, dx$

12. Set up, but do not evaluate, an integral for the area of the surface obtained by rotating the curve

$$y = e^x, \qquad 0 \leq x \leq 2$$

about the x-axis.

A.) $\int_0^2 2\pi e^x \sqrt{1+x^2}\, dx$

B.) $\int_0^2 2\pi x \sqrt{1+e^{2x}}\, dx$

C.) $\int_0^{2\pi} x \sqrt{1+x^2}\, dx$

D.) $\int_0^2 2\pi \sqrt{1+e^{2x}}\, dx$

E.) $\int_0^2 2\pi x \sqrt{1+x^2}\, dx$

F.) $\int_0^2 2\pi e^x \sqrt{1+e^{2x}}\, dx$

13. Find an equation for the tangent line to the curve

$$y = \sin(3t), \qquad x = e^t$$

at the point $(x, y) = (1, 0)$.

A.) $y = 3x - 3$
B.) $y = -3x + 3$
C.) $y = \frac{1}{3}x - \frac{1}{3}$
D.) $y = -x + 1$
E.) $y = x - 1$
F.) $y = -\frac{1}{3}x + \frac{1}{3}$

14. Find the area of the region enclosed by one loop of the curve $r = \sin 4\theta$.

A.) $\pi/16$ B.) $\pi/8$ C.) $\pi/4$ D.) $\pi/2$ E.) π F.) 2π

15. Consider the Cartesian coordinate $(x, y) = (2\sqrt{3}, -2)$. Find the polar coordinates (r, θ) of the point, where $r > 0$ and $0 \leq \theta < 2\pi$.

A.) $(r, \theta) = (2, \frac{7\pi}{6})$

B.) $(r, \theta) = (4, \frac{7\pi}{6})$

C.) $(r, \theta) = (2, \frac{\pi}{6})$

D.) $(r, \theta) = (2, \frac{5\pi}{6})$

E.) $(r, \theta) = (4, \frac{5\pi}{6})$

F.) $(r, \theta) = (4, \frac{11\pi}{6})$

16. Find the interval of convergence for the powers series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}.$$

A.) $(1,3)$ B.) $[1,3)$ C.) $(1,3]$ D.) $(-1,1)$ E.) $[-1,1)$ F.) $(-1,1]$

17. Find the Maclaurin series for $f(x) = x^2 e^{3x}$. What is the radius of convergence?

A.) $f(x) = \sum_{n=0}^{\infty} \frac{3^n x^{n+2}}{n!}$ with radius of convergence 1
B.) $f(x) = \sum_{n=0}^{\infty} \frac{3^n x^{n+2}}{n!}$ with radius of convergence 3
C.) $f(x) = \sum_{n=0}^{\infty} \frac{3^n x^{n+2}}{n!}$ with radius of convergence ∞
D.) $f(x) = \sum_{n=0}^{\infty} \frac{(3x)^{n+2}}{n!}$ with radius of convergence 1
E.) $f(x) = \sum_{n=0}^{\infty} \frac{(3x)^{n+2}}{n!}$ with radius of convergence 3
F.) $f(x) = \sum_{n=0}^{\infty} \frac{(3x)^{n+2}}{n!}$ with radius of convergence ∞

18. Does the series

$$\sum_{n=1}^{\infty} \frac{3 + \cos n}{2^n}$$

converge or diverge? Justify your answer.

19. Find the Maclaurin series, and radius of convergence for

$$g(x) = \frac{x^3}{(1-x)^2}\,.$$

Justify your answer.

20. Consider the series

$$\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}\,.$$

Does the series converge or diverge? Justify your answer.