MATH 104 – Final Exam - Fall 2014

1. Evaluate the integral $\int_{\pi}^{3\pi/2} 99 \sin^8 x \cos^3 x \, dx$

(a) 3 (b) 2 (c) 1 (d) 0 (e) -1 (f) -2

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2. The region in the first quadrant between the curve $y = \sqrt{16 - x^4}$, the x-axis ($y = 0$) and the y-axis ($x = 0$) is revolved around the y-axis. What is the volume of the resulting solid?

(a) $16\pi^2$ (b) $8\pi^2$ (c) $6\pi^2$ (d) $4\pi^2$ (e) $2\pi^2$ (f) π^2

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3. Find the value of $\int_0^{\infty} \frac{1}{x^2 - 2x + 2} \, dx$.

(a) diverges (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$ (e) π (f) 1

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4. Find the y-coordinate of the centroid of the region bounded by the curves $y = x$ and $y = x^2$.

(a) $-\frac{2}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{12}$ (d) $-\frac{1}{2}$ (e) $\frac{1}{2}$ (f) $\frac{1}{4}$

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5. Find the value of c so that the function $f(x) = c(1 - x^2)$ is a probability density function on $[-1, 1]$. What is its mean μ?

(a) $c = \frac{3}{4},\ \mu = 0$ (b) $c = \frac{3}{4},\ \mu = 1$ (c) $c = \frac{1}{2},\ \mu = 1$

(d) $c = \frac{3}{2},\ \mu = 0$ (e) $c = \frac{3}{2},\ \mu = 1$ (f) $c = \frac{1}{2},\ \mu = 0$

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6. Find the length of the curve $f(x) = \int_0^x \sqrt{3t+2}\,dt$ between $x = 0$ and $x = 2$.

(a) $2(9+\sqrt{3})$ (b) $2(9-\sqrt{3})$ (c) $\frac{8\sqrt{3}}{9}$

(d) $\frac{2}{3}(9-\sqrt{3})$ (e) $\frac{2}{3}(9+\sqrt{3})$ (f) $\frac{4\sqrt{2}}{3}$

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7. Determine if the series below converge absolutely $\mathcal{A}$, conditionally $\mathcal{C}$, or diverge $\mathcal{D}$.

I. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ II. $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n!}$

(a) $\mathcal{D}, \mathcal{C}$ (b) $\mathcal{D}, \mathcal{A}$ (c) $\mathcal{C}, \mathcal{D}$ (d) $\mathcal{C}, \mathcal{C}$
(e) $\mathcal{C}, \mathcal{A}$ (f) $\mathcal{A}, \mathcal{D}$ (g) $\mathcal{A}, \mathcal{C}$ (h) $\mathcal{A}, \mathcal{A}$

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8. Find the interval of convergence of the power series $\sum_{n=2}^{\infty} \frac{(x-3)^n}{3^n \ln n}$.

(a) $[0, 6)$ (b) $[0, 6]$ (c) $(0, 3]$ (d) $[0, 3]$ (e) $[-3, 9)$ (f) $[-3, 6]$

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9. Let $P_2(x)$ be the Taylor polynomial of degree two for $f(x) = \sqrt{x}$ centered at $a = 4$. What is the value of $P_2(5)$?

(a) $\sqrt{5}$ (b) $\frac{143}{64}$ (c) $\frac{73}{32}$ (d) $\frac{71}{32}$ (e) $\frac{35}{16}$ (f) $\frac{9}{4}$

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10. Find $y(2)$ if y satisfies the initial-value problem $xy' = x^3 - 2y$ and $y(1) = 1$.

(a) $\frac{9}{5}$ (b) $\frac{5}{6}$ (c) 2 (d) $\frac{4}{5}$ (e) -1 (f) $\frac{1}{6}$

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11. A tank initially contains 100 gallons of brine with 60 lbs of dissolved salt. Brine that contains 3 lbs of salt per gallon enters the tank at the rate of 3 gallons per minute and the well-stirred mixture leaves at the rate of 3 gallons per minute. Let $y(t)$ be the amount of salt in the tank at time t. What is $y(100 \ln 2)$?

(a) 30 (b) 80 (c) 260 (d) 270 (e) 300 (f) 320

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12. Evaluate $\lim_{x\to 0} \dfrac{x^2 e^x}{\cos x - 1}$

(a) -2 (b) -1 (c) $-\dfrac{1}{2}$ (d) 0 (e) $\dfrac{1}{2}$ (f) 2

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13. Compute $\displaystyle\int_1^\infty xe^{-2x}\,dx$.

(a) $\dfrac{1}{2e^2}$ (b) $\dfrac{3}{4e^2}$ (c) $\dfrac{2}{e^2}$ (d) $-\dfrac{2}{e^2}$ (e) $\dfrac{1}{4e^2}$ (f) The integral diverges

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14. If $\dfrac{x^4}{(x+1)(x^2+5)} = Ax + B + \dfrac{C}{x+1} + \dfrac{Dx+E}{x^2+5}$, find the value of C

(a) 0 (b) $\dfrac{1}{6}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{1}{2}$ (e) 2 (f) 4

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15. For (precisely) which values of p does $\displaystyle\sum_{n=0}^{\infty} \frac{2^{np}}{3^n + 4^n}$ converge?

(a) $p < 2$ (b) $p > 1$ (c) $p > \frac{1}{2}$ (d) $p < -1$ (e) $p > -2$ (f) $p < 0$

Answers (maybe): FDDBA — DEABA — DABBA