University of Pennsylvania
Math 104 Final Exam
Spring 2014

Name ____ **(print)** **Penn ID#** ________________

Professor____ **Recit. Number** ______________________

There are fifteen questions on this examination. No calculators are allowed, but you may use one standard sized 8.5”X11” sheet with notes handwritten on both sides. Show your work in the space provided, and then **transfer your answers** *carefully* to this sheet.
It is important to show your work because we will be going back over it – you might gain additional partial credit for substantial progress toward the solution of a problem, or you might *lose* credit for an unsubstantiated correct answer.
Please put away and silence (don’t set to vibrate) all electronic devices (computers, tablets, cell phones, mp3 players), use of these are forbidden during the examination period. Do your best!

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination. In particular, all the work on this test is my own.

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Signature

1.	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**	**9.**	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**
2.	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**	**10.**	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**
3.	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**	**11.**	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**
4.	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**	**12.**	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**
5.	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**	**13.**	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**
6.	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**	**14.**	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**
7.	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**	**15.**	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**
8.	**(A)**	**(B)**	**(C)**	**(D)**	**(E)**	**(F)**	**(G)**	**(H)**									

1. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = e^x$, $y = 0$, $x = 0$, and $x = 2$ about the line $x-\text{axis}$.

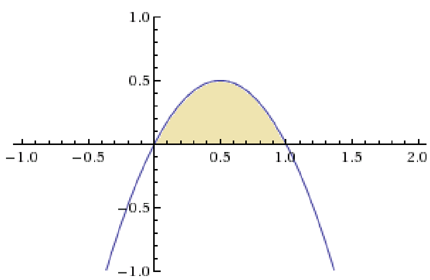
(A) $\frac{\pi}{4}e^2$ (E) $\frac{\pi}{2}\left(e^4 - 1\right)$

(B) $\frac{\pi}{2}e^4$ (F) $2\pi\left(e^4 - 1\right)$

(C) $2\pi e^2$ (G) $2\pi e^4$

(D) $\frac{\pi}{4}e$ (H) 2π

2. Find the volume of the solid generated by revolving the region bounded above by the graph of $y = 2x - 2x^2$ and below by the $x-\text{axis}$ about the line $x = 2$.



(A) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

(B) $\frac{\pi}{6}$ (F) $\frac{\pi}{2}$

(C) $\frac{2\pi}{3}$ (G) 2π

(D) $\frac{3\pi}{4}$ (H) π

3. Find the arclength of the curve $y = \frac{2\sqrt{3}}{9}\left(3x^2 + 1\right)^{3/2}$ from $x = -1$ to $x = 2$.

(A) 8 (E) 6

(B) 2 (F) 21

(C) 9 (G) 24

(D) 4 (H) 27

4. Evaluate the integral below

$$\int_{e}^{e^3} x\ln(x)\,dx$$

(A) $\frac{\pi}{4}e^2$

(B) $\frac{e^2}{4}$

(C) $\frac{e^2}{4}(3e^4-1)$

(D) $\frac{e^2}{4}(5e^4-1)$

(E) $\frac{e^2}{4}(7e^4-1)$

(F) $\frac{e^2}{4}(9e^4-1)$

(G) $\frac{7e^4}{4}$

(H) $\frac{7e^6}{4}$

5. Find the average value of $f(x)=\sin(x)\cdot\cos^4(x)$ over the interval $[0,\pi]$.

(A) $\frac{2}{5\pi}$

(B) $\frac{3}{5\pi}$

(C) $\frac{4}{5\pi}$

(D) $\frac{2}{3\pi}$

(E) $\frac{1}{3\pi}$

(F) $\frac{1}{4\pi}$

(G) $\frac{3}{4\pi}$

(H) π

6. Evaluate the integral below

$$\int_{0}^{2\sqrt{2}} \frac{x^2}{\left(16-x^2\right)^{3/2}}\,dx$$

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) $\frac{2}{3\pi}$

(E) $\frac{1}{3\pi}$

(F) $1-\frac{\pi}{4}$

(G) $\frac{3}{4\pi}$

(H) π

7. Find the area of the region enclosed by the graphs of $y=\dfrac{1}{x+1}$ and $y=\dfrac{1}{x+2}$ on the interval $[0,\infty)$.

(A) $\ln 6$ (E) $\sqrt{2}$

(B) $\ln 2$ (F) $\dfrac{1}{2}$

(C) 0 (G) 1

(D) π (H) ∞

8. Solve the initial value problem

$$5x\frac{dy}{dx}=y^2\ln x,\quad y(1)=2.$$

Find $y(e)$.

(A) 1 (E) $\dfrac{1}{2}$

(B) 2 (F) $\dfrac{3}{2}$

(C) 3 (G) $\dfrac{5}{2}$

(D) 4 (H) $\dfrac{7}{2}$

9. Find the solution to the differential equation

$$y'+\left[\frac{-3}{x(x-3)}\right]y=x-3,\quad x>3$$

satisfying $y(4)=2$.

(A) $y=\dfrac{x^3-3x^2}{8}$ (E) $y=\dfrac{x^2-3x}{2}$

(B) $y=e^{x^2}-e^{4x}+2$ (F) $y=\dfrac{x^2-x}{6}$

(C) $y=\dfrac{\ln(x)-x}{2}$ (G) $y=\dfrac{x^3+x}{34}$

(D) $y=\dfrac{\sin x+\cos x}{2}$ (H) $y=\dfrac{\sqrt{x}+x^{3/2}}{4}$

10. Determine the limit of the sequence

$$a_n = \left\{ \frac{n^2}{2n+1} - \frac{n^2}{2n-1} \right\}$$

(A) $\frac{-1}{2}$ (E) $\frac{1}{4}$

(B) $\frac{-1}{4}$ (F) $\frac{1}{2}$

(C) 0 (G) 1

(D) $\frac{1}{8}$ (H) ∞

11. The area inside the fractal known as the Koch snowflake can be described as the sum of the areas of infinitely many equilateral triangles. See the figure. For all but the center (largest) triangle, a triangle in the Koch snowflake is $\frac{1}{9}$ the area of the next largest triangle in the fractal. Suppose the largest (center) triangle has an area of 1 square unit. Then the area of the snowflake is given by the series

$$A = 1 + \frac{1}{3}\left(1 + \frac{4}{9} + \left(\frac{4}{9}\right)^2 + \left(\frac{4}{9}\right)^3 + \left(\frac{4}{9}\right)^4 + \cdots\right)$$

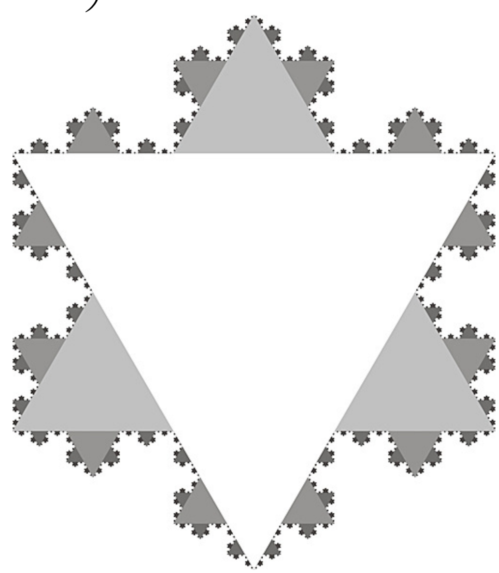
Find the area of the Koch snowflake.

(A) 6 (E) $\frac{8}{5}$

(B) 5 (F) $\frac{27}{5}$

(C) $\frac{5}{9}$ (G) $\frac{32}{5}$

(D) $\frac{4}{9}$ (H) ∞



http://www.behance.net/gallery/Worlds-Largest-Fractal-Vectors/720515

12. Determine whether the following series are convergent or divergent. For full credit be sure to explain your reasoning and tell what test was used.

$$(\mathrm{I})\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{\sqrt[5]{n^2}} \qquad (\mathrm{II})\sum_{n=1}^{\infty}\left(\frac{\pi}{\sqrt{2}}\right)^n \qquad (\mathrm{III})\sum_{n=1}^{\infty}\frac{3^{1/n}}{n^3}$$

	(I)	(II)	(III)
(A)	convergent	convergent	convergent
(B)	convergent	convergent	divergent
(C)	convergent	divergent	convergent
(D)	convergent	divergent	divergent
(E)	divergent	convergent	convergent
(F)	divergent	convergent	divergent
(G)	divergent	divergent	convergent
(H)	divergent	divergent	divergent

13. The interval of convergence of the power series

$$\sum_{n=1}^{\infty}\frac{2^{n+1}(x+3)^n}{3^n(n+1)^2}$$

(A) $\left(\frac{-9}{2},\frac{-3}{2}\right]$ (E) $(-5,-1]$

(B) $\left[\frac{-9}{2},\frac{-3}{2}\right)$ (F) $[-5,-1]$

(C) $\left(\frac{-9}{2},\frac{-3}{2}\right)$ (G) $\{-3\}$

(D) $\left[\frac{-9}{2},\frac{-3}{2}\right]$ (H) $(-\infty,\infty)$

14. Let $f(x)=\dfrac{1}{\sqrt{x}}$. The second order Taylor polynomial (quadratic) approximation centered at $x=4$ is:

(A) $\frac{1}{2}-\frac{1}{16}x+\frac{3}{256}x^2$

(B) $\frac{3}{2}-\frac{1}{12}x+\frac{5}{128}x^2$

(C) $\frac{1}{2}-\frac{1}{16}(x-4)+\frac{3}{256}(x-4)^2$

(D) $\frac{3}{2}-\frac{1}{12}(x-4)+\frac{5}{128}(x-4)^2$

(E) $\frac{1}{2}-\frac{3}{8}(x-4)+\frac{5}{32}(x-4)^2$

15. Use an appropriate Maclaurin series to estimate

$$\int_0^1 x\cos\left(\sqrt{x}\right)dx$$

with error less than 0.01. Explain.

(A) $\frac{51}{155}$

(B) $\frac{11}{32}$

(C) $\frac{17}{36}$

(D) $\frac{25}{96}$

(E) $\frac{101}{134}$

(F) $\frac{25}{38}$

(G) $\frac{11}{24}$

(H) $\frac{27}{92}$

ANSWERS:

1. E
2. H
3. F
4. D
5. A
6. F
7. B
8. G
9. E
10. A
11. G
12. C
13. D
14. C
15. B