Math 104 Make-up Final Exam — Spring 2010

1. Compute the total area bounded by the curves $y = x$ and $y = x^3$.

(a) $3/4$ (b) $2/3$ (c) $1/2$ (d) $1/3$ (e) $1/5$ (f) 0

2. Consider the the region bounded by the curve $y = \sqrt{x}$ and the lines $x = 1$ and $y = 0$. Find the volume of the solid obtained by rotating this region about the y-axis.

(a) $3\pi/4$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/5$ (e) $3\pi/5$ (f) $4\pi/5$

3. Find the volume obtained by rotating the solid bounded by the curves $y = x^2$ and $y = x^3$ about the x-axis.

(a) $\pi/7$ (b) $2\pi/7$ (c) $\pi/5$ (d) $4\pi/5$ (e) $2\pi/35$ (f) $4\pi/35$

4. Evaluate $\displaystyle\int_0^1 \left(\pi \sin \pi x - \frac{1}{x^2+1}\right) dx$.

(a) $2 - \frac{\pi}{4}$ (b) $13\frac{1}{4} + \ln 4$ (c) $e^3 - 2^{\frac{1}{4}}$ (d) $\cos\frac{4}{3} - \ln 2$ (e) $172\frac{1}{2} - \sqrt{3}$ (f) 1.2146

5. Evaluate $\displaystyle\int_1^e \frac{\ln x}{x^2}\, dx$.

(a) $e - e^{-2}$ (b) $1 - \frac{2}{e}$ (c) $\frac{\ln e^2}{e^4}$ (d) $1 - \frac{3}{e}$ (e) $\sqrt{\ln 2} - 1$ (f) 2^e

6. Evaluate $\displaystyle\int_0^1 \frac{dx}{\sqrt{4 - x^2}}$.

(a) 0 (b) 1 (c) $1 + \pi$ (d) $\pi/6$ (e) $\pi/2$ (f) π^2

7. Evaluate $\displaystyle\int_0^2 \frac{4 - 2x}{(x+2)(x^2+4)}\, dx$.

(a) $\pi/4 - 1/2$ (b) $\frac{1}{2}\ln 2$ (c) $32/51$ (d) $\pi/8$ (e) $e/9$ (f) $\cos 2$

8. Evaluate the improper integral $\displaystyle\int_0^2 \frac{1}{(x-1)^{\frac{4}{3}}}\, dx$.

(a) $\frac{2}{3} - \pi/2$ (b) $1/\ln 2$ (c) 1.442 (d) $1 - e$ (e) $\pi - 2$ (f) The integral is divergent.

9. Find the arclength of the part of the curve $x = \dfrac{1}{3}(y^2 + 2)^{\frac{3}{2}}$ between the points $(\sqrt{3}, 1)$ and $(2\sqrt{6}, 2)$.

(a) $10/3$ (b) $7/3$ (c) 2 (d) $14/3$ (e) 1 (f) $5/3$

10. Which of the following integrals corresponds to the surface area of revolution obtained by rotating the graph of $y = e^x$, from $x = 0$ to $x = 1$, about the x-axis?

(a) $\displaystyle\int_0^1 \pi e^{2x}\sqrt{1+e^x}\, dx$ (b) $\displaystyle\int_0^1 \pi x e^x \sqrt{e^x + e^{2x}}\, dx$ (c) $\displaystyle\int_0^1 2\pi e^x \sqrt{1 + e^x}\, dx$

(d) $\displaystyle\int_0^1 2\pi e^x \sqrt{1 + e^{2x}}\, dx$ (e) $\displaystyle\int_0^1 \pi e^{2x}\sqrt{e^x - 1}\, dx$ (f) $\displaystyle\int_0^1 \pi e^{x+1}\sqrt{e^x - e^{-x}}\, dx$

11. What is the average value of the function $f(x) = \sin x$ between $x = 0$ and $x = \pi$?

(a) 0 (b) 1 (c) 1/2 (d) $\pi/6$ (e) $2/\pi$ (f) $1/\sqrt{2}$

12. Consider the initial value problem $\frac{dy}{dt} = \frac{t}{y}$, with $y(0) = -3$. Find $y(4)$.

(a) -3 (b) 3 (c) -5 (d) 5 (e) -7 (f) 7

13. A population is observed to obey the logistic equation $\dfrac{dP}{dt} = 2P\left(1 - \dfrac{P}{1000}\right)$. If $P(0) = 500$, when does the population reach 2000?

(a) $t = 1$ (b) $t = 2$ (c) $t = 10$ (d) $t = 500$ (e) $t = 1000$ (f) never.

14. Determine if the sequence $a_n = (n + e^n)^{1/n}$ converges or diverges. If it converges, find the limit.

(a) 1 (b) 2 (c) e (d) $1 + e$ (e) 3 (f) The sequence is divergent.

15. Determine if the series $\displaystyle\sum_{n=0}^{\infty} \frac{3 \cdot 2^{n+1}}{5^n}$ converges or diverges. If it converges, find the sum.

(a) 2 (b) 3 (c) 5 (d) 10 (e) 15 (f) The series is divergent.

16. Determine all real numbers r for which the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^r + \ln n}$ converges.

(a) $r > -1$ (b) $r > 0$ (c) $r > 1/2$ (d) $r > 1$ (e) $r > 3/2$ (f) $r > 2$

17. Examine the two series below for absolute convergence (A), conditional convergence that is not absolute (C), or divergence (D).

$$(1) \quad \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)} \qquad (2) \quad \sum_{n=1}^{\infty} (-1)^{n-1} 2^{-n}$$

(a) 1C, 2A (b) 1A, 2C (c) 1A, 2D (d) 1A, 2A (e) 1C, 2C (f) 1C, 2D

18. Find the interval of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2n+1}$.

(a) $[-1, 1]$ (b) $[-1, 1)$ (c) $(-1, 1)$ (d) $(-1, 1]$ (e) $(-2, 2)$ (f) $(-2, 2]$

19. The Maclaurin series for the function $\dfrac{x^2}{1 - x^3}$ is

(a) $1 - x + x^2 - x^3 + \cdots$. (b) $x + \dfrac{3x^2}{2!} + \dfrac{4x^3}{3!} - \dfrac{5x^4}{4!} + \cdots$.

(c) $x^2 - x^3 + x^4 - x^5 + \cdots$. (d) $x^2 + x^5 + x^8 + x^{11} + \cdots$.

(e) $x^{2!} + x^{3!} + x^{4!} + x^{5!} + \cdots$. (f) $x^3 + x^5 + x^7 + x^9 + \cdots$.

20. The coefficient of $(x - 1)^{10}$ in the Taylor series for the function e^x at $x = 1$ is

(a) 1. (b) $\dfrac{1}{10!}$. (c) $\dfrac{e^{10}}{10!}$. (d) $\dfrac{e^{10}}{10}$. (e) $\dfrac{e}{10!}$. (f) e^{10}.