

1. Find the arc length of the curve $y = \frac{1}{4}(\sin x + \cos x)$ for $0 \leq x \leq \frac{\pi}{2}$.

- a) $1 + \frac{1}{\sqrt{2}}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{2}$ d) 1 e) 2

2. Compute

$$\int_0^{\pi} x \sin 2x \, dx .$$

- a) $-\frac{\pi}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{2} + \frac{1}{2}$ d) $\frac{\pi}{2} - \frac{1}{2}$ e) 0

3. The base of a solid is the area between the curve $y = x^2$, for $0 \leq x \leq 1$, and the x -axis. The cross sections perpendicular to the x -axis are equilateral triangles. Find the area of this solid.

- a) $\frac{1}{5}$ b) $\frac{3}{20}$ c) $\frac{\sqrt{3}}{12}$ d) $\frac{1}{4}$ e) $\frac{\sqrt{3}}{20}$

4. Find: $\int_1^{\infty} \frac{1}{1+x^2} \, dx$.

- a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) $\frac{3\pi}{3}$ d) π e) diverges

5. Which of the following series converges:

I. $\sum_3^{\infty} n \sin^2 \frac{1}{n}$, II. $\sum_1^{\infty} \frac{ne^n}{3^n}$ III. $\sum_1^{\infty} n \sin \frac{1}{n}$,

- a) None b) I and II c) I and III d) Only II e) All three

6. What is the interval of convergence of

$$\sum_{n=0}^{\infty} \frac{n(2x-3)^n}{n^2+1} .$$

- a) $1 < x < 2$ b) $1 \leq x < 2$ c) $1 < x \leq 2$ d) $1 \leq x \leq 2$ e) all x .

7. The bounded region in the first quadrant between the curves $y = x^2$ and $y = 2 - x$ is rotated around the x -axis. Find the volume.

- a) 2π b) $\frac{7\pi}{3}$ c) $\frac{13\pi}{15}$ d) $\frac{9\pi}{5}$ e) $\frac{32\pi}{15}$.

8. Compute the integral

$$\int_0^{\frac{\pi}{4}} \sin^2 x \cos^3 x \, dx .$$

- a) $\frac{7}{60\sqrt{2}}$ b) $\frac{1}{12\sqrt{2}}$ c) $\frac{2}{30}$ d) $-\frac{1}{40\sqrt{2}}$ e) 0.

9. The approximation

$$e \approx 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24}$$

is obtained from the first five terms of the MacLaurin expansion. From the Taylor remainder theorem, what is the guaranteed maximum absolute value of the error. (You may use $e \leq 3$.)

- a) $\frac{1}{8}$ b) $\frac{1}{24}$ c) $\frac{1}{120}$ d) $\frac{1}{40}$ e) $\frac{1}{6}$.

10. A certain medical study asserts that the number y of brain cells a person loses per day is related to the number of ounces x of alcohol consumed per day by the differential equation

$$\frac{dy}{dx} = 2xy \ln 10.$$

Assuming a person who does not drink alcohol loses 1000 brain cells per day, how much will a person who consumes two ounces of alcohol lose.

- a) 10^4 b) 10^5 c) 10^6 d) 10^7 e) 10^8 .

11. The third non-vanishing (i.e. non-zero) term in the MacLaurin expansion of the function

$$f(x) = \int_0^x \cos^2 t \, dt$$

is

- a) 0 b) $\frac{x^5}{15}$ c) $\frac{4x^3}{3}$ d) $-\frac{x^3}{3}$ e) $\frac{x^4}{3}$.

12. The series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^{\frac{4}{5}} - 1}$$

a) converges absolutely b) diverges c) behavior cannot be determined d) converges conditionally e) would converge if the sum began with $n = 3$.

13. Determine the limit of the sequence

$$\frac{3^n}{\sqrt{n}},$$

if it exists.

- a) 0 b) 1 c) 2 d) e e) does not exist.

14. Use Euler's method with step size one to determine $y(2)$ if $y(0) = 1$ and

$$\frac{dy}{dx} = 2xy^2.$$

- a) 0 b) 1 c) 2 d) 3 e) 4.

15. The radius of convergence of the series

$$\sum_{n=1}^{\infty} n^{\frac{3}{4}} (7x - 2)^n$$

is

- a) 0 b) $\frac{1}{7}$ c) $\frac{2}{7}$ d) $\frac{3}{7}$ e) ∞ .

16. Find the total area of the region bounded by the curves $y = x$ and $y = x^5$.

- a) 0 b) $\frac{1}{6}$ c) $\frac{1}{3}$ d) $\frac{2}{3}$ e) 1.

17. Solve the differential equation

$$\frac{dy}{dx} = \frac{e^x}{4y^3},$$

subject to the initial condition $y(0) = 2$.

a) $y = e^{\frac{x}{4}} + 1$ b) $y = (e^x + 7)^{\frac{1}{4}}$ c) $y = (e^x + 7)^{\frac{1}{3}}$ d) $y = (e^x + 31)^{\frac{1}{5}}$ e) $y = (e^x + 15)^{\frac{1}{4}}$.

18. Compute the integral

$$\int_{\sqrt{2}}^2 \frac{dx}{\sqrt{x^2 - 1}}.$$

a) 1 b) $\ln \frac{1+\sqrt{2}}{2+\sqrt{3}}$ c) $\ln \frac{\sqrt{3}+\frac{2}{\sqrt{3}}}{1+\sqrt{2}}$ d) $\ln \frac{2+\sqrt{2}}{1+\sqrt{3}}$ e) $\ln \frac{2+\sqrt{3}}{1+\sqrt{2}}$.

19. Compute the integral

$$\int_0^1 \frac{dx}{x - \frac{1}{2}}.$$

a) divergent b) 0 c) $2 \ln 2$ d) $-2 \ln 2$ e) $\frac{1}{2} \ln 2$.

20. Compute the integral

$$\int_3^4 \frac{dx}{x^2 - 2x}.$$

a) $\frac{2}{3}$ b) $\ln \frac{2}{3}$ c) $\frac{3}{2}$ d) $\ln \frac{3}{2}$ e) $\frac{1}{2} \ln \frac{3}{2}$.

Answer Key

1.d 2.a 3.e 4.b 5.d 6.b 7.e 8.a 9.d 10.d 11.b 12.d 13.a 14.d 15.b 16.c 17.e 18.e 19.a 20.e