# MATHEMATICS DEPARTMENT
# UNIVERSITY *of* PENNSYLVANIA
## Mathematics 104 Final Examination
## Spring 2009

*Your Name:*________________________________________

*Your Professor (check one):* ❑ *Crotty* ❑ *Shaneson* ❑ *vanErp*

*Your TA:* ____________________________________________________

**Instructions:**

**You have 2 hours to complete this examination.**

***Please write all answers below***

- This exam contains 20 multiple-choice questions. Please write the letter corresponding to your answer on the appropriate line *on your answer sheet (below)*. If you change an answer, please either erase or cross out the answer you do not want considered; questions with more than one answer will be marked wrong.
- Please show your work in the space provided on this question sheet. An answer with no supporting work may receive little or no credit. Partial credit *may* be given based on your work.
- Each question is worth 10 points.

**Answers:**

| | | | |
|---|---|---|---|
| **1** | **C** | **11** | **E** |
| **2** | **F** | **12** | **B** |
| **3** | **A** | **13** | **F** |
| **4** | **B** | **14** | **C** |
| **5** | **D** | **15** | **B** |
| **6** | **C** | **16** | **C** |
| **7** | **E** | **17** | **A** |
| **8** | **A** | **18** | **E** |
| **9** | **B** | **19** | **F** |
| **10** | **E** | **20** | **D** |

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------------------------------------------*Please do not write below this line*------------------------------------------

**Score:** **Notes:**

1. Find the arc length of the graph of $y = x^{\frac{3}{2}}$ for $0 \le x \le 4$.

a) $\frac{928}{135}$ b) $\frac{64}{27}$ c) $\frac{8}{27}\left(10\sqrt{10} - 1\right)$ d) 2 e) $\frac{94}{27}\sqrt{94} - \frac{13}{27}\sqrt{13}$ f) 7

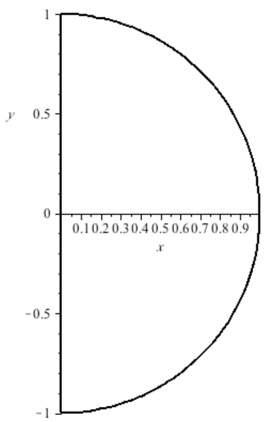
2. Compute: $\int_0^\pi x\sin x \; dx$

a) 4π – 2 b) 2π – 2 c) π/2 d) 4π – 4 e) π – 2 f) π

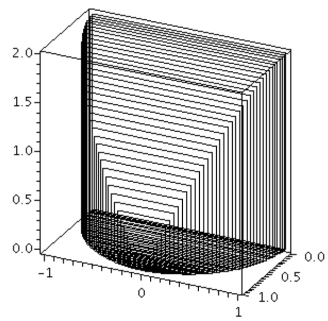
3. The base of a solid is a semi-circular disk $\left\{(x, y) \middle| x^2 + y^2 \leq 1,\ x \geq 0\right\}$. Cross sections perpendicular to the $x$-axis are squares with their vertices on the semi-circle. Compute the volume of the solid.

a) $\dfrac{8}{3}$ b) $\pi^2$ c) $\dfrac{2\pi}{3}$ d) $\dfrac{\pi^2}{4}$ e) 1 f) 4

The Base




The Solid



4. Compute: $\int_1^{\infty} \frac{1}{(1+x)^4}\, dx$

a) 0 b) $\frac{1}{24}$ c) $\frac{1}{8}$ d) $-\frac{1}{4}$ e) $\frac{1}{2}$ f) divergent

5. Which of the three series given below converge?

I) $\sum_{n=1}^{\infty} \frac{n}{n+1}$ II) $\sum_{n=0}^{\infty} \frac{\pi^n}{3^n}$ III) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

a) all diverge b) I only c) II only d) III only

e) I and II only f) all converge

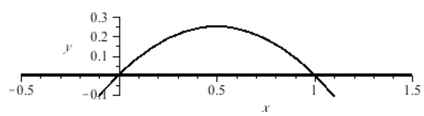
6. What is the radius of convergence of the series $\sum_{n=0}^{\infty} n(n+1)^2(\pi x - 2)^n$

a) $\pi$ b) $\frac{2}{\pi}$ c) $\frac{1}{\pi}$ d) 0 e) 2 f) $\infty$

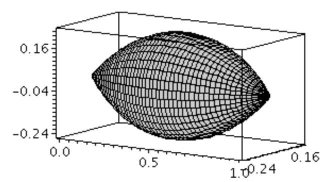
7. The region bounded by the curves $y = x - x^2$ and $y = 0$ is rotated about the $x$-axis. Compute the volume of the resulting solid.

a) $\dfrac{\pi}{3}$ b) $\dfrac{\pi}{6}$ c) $\dfrac{\pi}{12}$ d) $\dfrac{\pi}{15}$ e) $\dfrac{\pi}{30}$ f) $\dfrac{\pi}{36}$

**The region**




**The solid**



8. Compute: $\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx$

a) $\frac{2}{15}$ b) $\frac{4}{15}$ c) $\frac{2}{5}$ d) $\frac{8}{15}$ e) $\frac{14}{15}$ f) divergent

9. The approximation $e \approx 1 + 1 + \frac{1}{2} + \frac{1}{6} = \frac{8}{3}$ is obtained from the first four terms of the Maclaurin expansion of $e^x$, $x = 1$. From the Taylor Remainder Theorem, which of the following is the *guaranteed* maximum absolute value of the error, $|\mathbf{R_n}|$, of this approximation (you may take it as known that $e \leq 3$).

a) $\frac{1}{24}$ b) $\frac{1}{8}$ c) $\frac{1}{6}$ d) 0 e) $\infty$ f) 1

10. Market research has shown the price ($p$) and weekly sales ($y(p)$ ) of a particular product are related by the following differential equation:

$$\frac{dy}{dp} = -\frac{1}{2}\left(\frac{y}{p+3}\right)$$

If sales amount to 100 units when the price is \$1 (i.e., y(1) = 100), what will the weekly sales be if the price is raised to \$6?

a) 0 b) 100/18 c) 100/6 d) 50 e) 200/3 f) 100

11. The third non-vanishing (i.e., non-zero) term in the Maclaurin expansion of the function:

$$f(x) = \int_0^x \sin t^2 \, dt$$

is:

a) $\dfrac{x^{10}}{10!}$ b) $\dfrac{x^{11}}{120}$ c) $-\dfrac{x^{11}}{1320}$ d) $\dfrac{x^{11}}{11!}$ e) $\dfrac{x^{11}}{1320}$ f) $\dfrac{x^{9}}{12}$

12. The series $\sum_{n=0}^{\infty}(-1)^n \frac{1}{1+\sqrt{n}}$

a) converges absolutely b) converges conditionally c) diverges

d) convergence/divergence cannot be determined

13. Determine the limit of the sequence $\frac{(-2)^n}{n}$ if it exists.

a) 0 b) ln2 c) 1 d) 2 e) $e$ f) divergent

14. Use Euler's method with a step size of 1 to estimate $y(2)$ if

$$\frac{dy}{dx} = xy^2 \text{ subject to } y(0) = 1$$

a) 0 b) 1 c) 2 d) 3 e) 4 f) $\infty$

15. Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{(-3)^n}{\sqrt{n+1}}(x-1)^n$

a) $\left(\frac{2}{3}, \frac{4}{3}\right)$

b) $\left(\frac{2}{3}, \frac{4}{3}\right]$

c) $\left[\frac{2}{3}, \frac{4}{3}\right)$

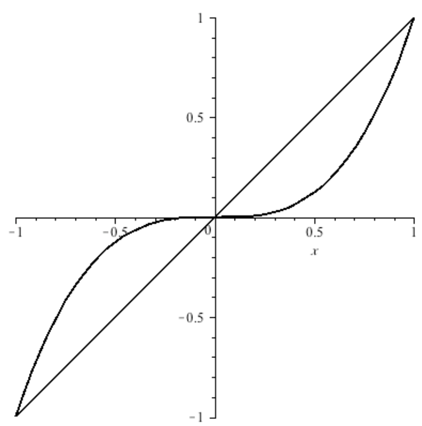
d) $\left(\frac{1}{3}, \frac{5}{3}\right)$

e) $\left[\frac{1}{3}, \frac{5}{3}\right)$

f) $\left(\frac{1}{3}, \frac{5}{3}\right]$

16. Find the total area of the region bounded by the curves $y = x$ and $y = x^3$.

a) 0 b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$ e) $\frac{3}{4}$ f) 1



17. Solve the differential equation $\dfrac{dy}{dx} = \dfrac{e^{2x}}{6y^5}$ subject to $y(0) = 1$.

a) $y = \pm\sqrt[6]{\dfrac{1}{2}e^{2x} + \dfrac{1}{2}}$

b) $y = \pm\sqrt[6]{\dfrac{e^{2x}}{2}}$

c) $y = \pm\sqrt[6]{2e^{2x}}$

d) $y = \pm\sqrt[5]{\dfrac{e^{2x}}{2}}$

e) $y = \pm\sqrt[6]{\dfrac{1}{5}e^{6x} + 5}$

f) $y = \pm\sqrt[6]{e^{2x} + 2}$

18. Compute: $\int_0^1 \frac{dx}{\sqrt{x^2+1}}$

a) $\pi$ b) $\frac{3}{4}$ c) $\ln\left(\frac{\sqrt{2}}{2}\right)$ d) $\frac{\pi}{4}$ e) $\ln\left(1+\sqrt{2}\right)$ f) 0

19. Compute; $\int_0^1 \frac{dx}{3x-2}$

a) 0 b) 2 c) $\frac{\ln 2}{3}$ d) $-\frac{\ln 2}{3}$ e) $\frac{\ln 3}{2}$ f) divergent

20. Compute: $\int_2^3 \frac{dx}{x^2 - x}$

a) $\frac{3}{2}$ b) $\frac{4}{3}$ c) $\ln 2$ d) $\ln \frac{4}{3}$ e) $\ln \frac{3}{2}$ f) $\ln 3$