UNIVERSITY *of* PENNSYLVANIA
MATHEMATICS DEPARTMENT
MATH 104
FINAL EXAMINATION/SPRING 2008

NAME: ____

YOUR PROFESSOR (CHECK ONE): ❑ACKERMAN ❑ CROTTY ❑ SHANESON

YOUR TA: ____

INSTRUCTIONS:
1. You have two hours for this examination.
2. You are permitted the use of a one page notes sheet (8.5x11, both sides).
3. Solve each problem in the space provided. Write the letter of your answer in the appropriate space on this page.
4. Show your work. A correct answer with no supporting work may receive little or no credit
5. Each problem is worth 4 points.
6. There are 25 problems; *you are to do all of them*.

Write the letters corresponding to your answers here:

1.	D
2.	A
3.	C
4.	B
5.	E
6.	C
7.	E
8.	A
9.	D
10.	C
11.	B
12.	E
13.	E
14.	C
15.	B
16.	E
17.	D
18.	C
19.	D
20.	C
21.	D
22.	A
23.	D
24.	H
25.	B

Score: ___________________

1. Find the volume of the solid obtained when the region bounded by $y = x^2 - x$, *and* $y = 0$ is rotated around the *x*-axis.

a) $\pi/3$ b) $\pi/5$ c) $\pi/12$ d) $\pi/30$ e) $\pi/36$

2. Suppose that $g(x)$ is the inverse of the function $f(\mathrm{x})$. $f(4) = 5$ and $f'(4) = 2/3$. Find $g'(5)$.

a) 3/2 b) 2/3 c) 1 d) 2 e) 3

3. Compute $f'(1)$ if $f(x) = e^{-x^2}$.

a) e b) 1 c) $-2e^{-1}$ d) 2e e) does not exist

4. Find the value of $\int_0^1 \frac{1}{\sqrt{4-x^2}} dx$

a) π/8 b) π/6 c) π/4 d) π/2 e) 0

5. Compute $\lim_{x\to 0^+} \dfrac{x}{\sin x + \tan x}$

a) -2 b) 0 c) $-1/2$ d) ¼ e) ½

6. Compute $\int_1^4 \sqrt{t}\ln t \; dt$.

a) 4ln4 b) $\dfrac{4}{3}\ln 4$ c) $\dfrac{16}{3}\ln 4 - \dfrac{28}{9}$ d) $12\ln 4 - \dfrac{25}{9}$

e) $\dfrac{22}{3}\ln 4 - 3$

7. Compute $\int_0^{\pi} \cos^2 x \sin x \; dx$

a) 0 b) 8π/3 c) 8/3 d) 4/3 e) 2/3

8. Compute $\int_0^1 \sqrt{2x - x^2} \; dx$.

a) π/4 b) π/2 c) ½ d) 1 e) 0

9. Compute $\int_0^1 \frac{2x^2}{(x+1)(x^2+1)}\,dx$

a) $5\sqrt{5}-1$ b) $3\pi/2$ c) $\frac{\pi}{2}\left(3\sqrt{3}-1\right)$ d) $\frac{3}{2}\ln 2-\frac{\pi}{4}$

e) 0

10. Evaluate $\int_{-\infty}^{0} e^{3x}\,dx$.

a) 3 b) 1 c) 1/3 d) 0 e) divergent

11. Find the arc length of the curve given by $y=\frac{2}{3}x^{\frac{3}{2}}$, $0 \le x \le 3$.

a) 13/3 b) 14/3 c) 5 d) 16/3 e) 17/3

12. Find the area of the surface generated when the curve $y=\sqrt{x}$, $1 \le x \le 2$ is rotated about the x-axis.

a) $\pi\frac{\left(6\sqrt{6}-1\right)}{3}$ b) $\pi\frac{\left(8-2\sqrt{2}\right)}{3}$ c) $\pi\frac{\left(5\sqrt{5}-2\sqrt{2}\right)}{3}$

d) $\pi\frac{\left(3\sqrt{3}-1\right)}{3}$ e) $\pi\frac{\left(27-5\sqrt{5}\right)}{6}$

13. Describe the curve defined by $x = \sin t$, $y = \sin^2 t$.

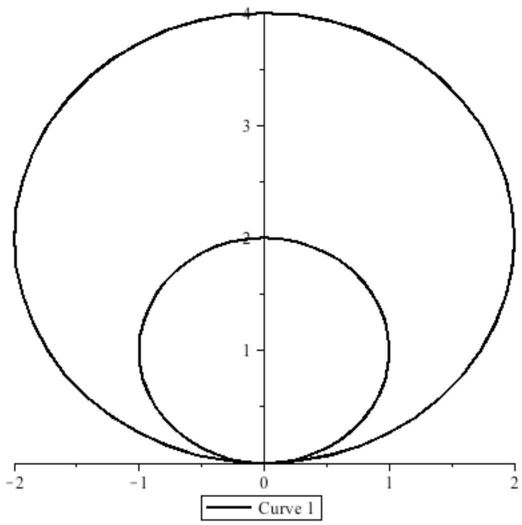
a) circle b) ellipse c) hyperbola d) one branch of a hyperbola
e) a part of a parabola

14. Find the slope of the line tangent to the curve defined by $x = 2\ln t$, $y = te^t$ when $t = 1$.

a) 1 b) 2 c) e d) π e) 4

15. Find the area of the region inside the curve $r = 4\sin\theta$, but not inside the curve $r = 2\sin\theta$.

a) 12π b) 3π c) π d) 4π e) $\pi/2$



16. Determine the limit of the sequence $\left\{\dfrac{(-2^n)}{n}\right\}_{n=1}^{\infty}$

a) -2 b) 0 c) $\ln 2$ d) e^2 e) divergent

17. Which of these three series converge?

1) $\sum_{n=1}^{\infty} \frac{n}{n+1}$ 2) $\sum_{n=1}^{\infty} \frac{\pi^n}{3^n}$ 3) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

a) none b) 1 c) 2 d) 3 e) 1,2
f) 1,3 g) 2,3 h) 1, 2, 3

18. Which of the following tests will establish that the series $\sum_{n=1}^{\infty} \frac{n}{\sqrt{2n^5+1}}$ converges?

1) Comparison test with $\sum_{n=1}^{\infty} n^{-5/2}$

2) Comparison test with $\sum_{n=1}^{\infty} n^{-3/2}$

3) Comparison test with $\sum_{n=1}^{\infty} n^{-1/2}$

a) none b) 1 c) 2 d) 3 e) 1, 2
f) 1, 3 g) 2, 3 h) 1, 2, 3

19. If we add the first 100 terms of the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\ldots$ how close is the partial sum s_{100} to the sum, s, of the series?

a) $s_{100} > s$ with $s_{100} - s < 1/101$
b) $s_{100} < s$ with $s_{100} - s < 1/101$
c) $s_{100} > s$ with $s_{100} - s < 1/100$
d) $s_{100} < s$ with $s_{100} - s < 1/100$
e) $s_{100} < s$ with $s_{100} - s < 1/e^{101}$
f) cannot be determined

20. Examine the series below for Absolute convergence, Conditional convergence or Divergence:

1) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(n+2)3^n}{2^{2n+1}}$ 2) $\sum(-1)^{n-1}\frac{(n+3)2^{2n}}{3^{n+100}}$

a) 1A, 2A b)1A, 2C c) 1A, 2D d) 1C, 2A e) 1C, 2C
f) 1C, 2D g) 1D, 2A h) 1D, 2C

21. What is the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x+3)^n}{2^n \sqrt{n}}$?

a) ½ b) 1 c) 3 d) 2 e) divergent

22. Find the coefficient of the x^2 term in the Maclaurin series for e^{x-1}.

a) 2 b) e c) e^2 d) 0 e) $\frac{1}{2e}$

23. Find a series representation of $\int \frac{e^x}{x}\,dx$.

a) $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)!} + C$

b) $\sum_{n=0}^{\infty} \frac{x^{n+1}}{n!} + C$

c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n} x^n + C$

d) $\ln|x| + \sum_{n=0}^{\infty} \frac{x^n}{n \bullet n!} + C$

e) $\ln|x| + \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n + C$

24, Suppose that you wish to estimate the value of $\int_a^b f(x)\,dx$ using Simpson's rule with "n" subintervals between *a* and *b*. By what factor is the upper bound of the error estimate increased/decreased if you then recalculate the estimate using 2*n* subintervals?

a) 2 b) ½ c) 4 d) ¼ e) 8

f) 1/8 g) 16 h) 1/16

25. Let $f(x) = \dfrac{\ln x}{x}$. Over which of the following open intervals is f always decreasing?

a) $(0, 1/e)$ b) (e, ∞) c) $(0, 1)$

d) $(0, 2)$ e) $(1/e, \infty)$