MATH 104 – Final Exam - Spring 2004

1. Calculate the area bounded by the curves $y = \dfrac{x}{x^2+1}$ and $y = x^2/2$.

   (a) 1 (b) $\dfrac{1}{2}\ln 2 - \dfrac{1}{6}$ (c) $\dfrac{1}{6}\ln 2 - \dfrac{1}{2}$ (d) $\dfrac{1}{2}\ln 6 - \dfrac{1}{6}$ (e) $\dfrac{1}{6}\ln 6 - \dfrac{1}{2}$

2. Calculate the volume of the solid obtained by rotating the region bounded by $y = e^{2x}$, and $y = e^{x^2}$ around the y-axis.

   (a) $\dfrac{\pi}{2}(e^2+e)$ (b) $\dfrac{\pi}{4}(e^4-2e)$ (c) $\dfrac{\pi}{2}(e^4+3)$

   (d) $\dfrac{\pi}{4}(e^4-2e^2-2)$ (e) $\dfrac{\pi}{2}(e^2-e)$

3. Calculate the length of the part of the curve $y = \dfrac{2}{3}x^{3/2}$ for $0 \le x \le 3$.

   (a) 14/3 (b) 16/5 (c) 19/3 (d) 21/5 (e) 26/3

4. Calculate the integral $\displaystyle\int_{1/2}^{1} \frac{1}{x^2\sqrt{1-x^2}}\,dx$

   (a) $2+\ln 2$ (b) $\dfrac{\sqrt{2}}{2}+1$ (c) $2-\ln(\sqrt{2}-1)$

   (d) $\sqrt{3}$ (e) diverges

5. Calculate the integral $\displaystyle\int_{1}^{3} \frac{x-1}{x^3+4x^2+3x}\,dx$

   (a) $\dfrac{1}{3}\ln 3 - \ln 2$ (b) $\dfrac{5}{3}\ln 2 - \dfrac{1}{3}\ln 3$ (c) $\dfrac{2}{3}\ln 3 + \dfrac{1}{2}\ln 2$

   (d) $\dfrac{5}{3}\ln 3 - \dfrac{3}{5}\ln 2$ (e) $\dfrac{5}{3}\ln 2 - \ln 3$

6. Calculate $\displaystyle\int_{0}^{1} \frac{1}{\sqrt{x}(1+x)}\,dx$

   (a) $\dfrac{\pi}{2}$ (b) $\dfrac{1}{\sqrt{2}}$ (c) $\dfrac{\pi}{4}-1$ (d) $\sqrt{2}-\dfrac{1}{\sqrt{2}}$ (e) diverges

7. If $y = f(x)$ is the solution of the initial-value problem $y' + xy = x$, $y(0) = 0$, then what is the value of $f(1)$?

   (a) $1-e$ (b) $1-\sqrt{e}$ (c) $1-\dfrac{1}{\sqrt{e}}$ (d) $1+\sqrt{3}$ (e) $2-\dfrac{1}{\sqrt{e}}$

8. Under just the right conditions, the thickness of a uniform sheet of melting ice will decrease at a rate proportional to the square of the thickness of the sheet. At 12 o'clock, the ice sheet was 1 cm thick, and at 1 o'clock, it was 0.5 cm thick. At what time will the ice sheet be 0.25 cm thick?

(a) 1:30 (b) 2:00 (c) 2:30 (d) 3:00 (e) 4:00

9. Calculate: $\displaystyle\int_1^\infty \frac{\ln x}{x^3}\,dx$.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{9}$ (e) diverges

10. What is the limit of the sequence $\left\{\left(1+\frac{1}{n^2}\right)^n\right\}$?

(a) 0 (b) 1 (c) e (d) e^2 (e) does not exist

11. The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n!}{2^n}$

(a) converges absolutely (b) converges conditionally (c) diverges

12. The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n^2+1}$

(a) converges absolutely (b) converges conditionally (c) diverges

13. For (precisely) which values of x does the power series $\displaystyle\sum_{n=2}^{\infty} \frac{n^2(x-3)^n}{3^n}$ converge?

(a) $0 < x < 6$ (b) $2 < x < 4$ (c) $-3 < x < 3$ (d) $0 \le x < 6$ (e) $-3 \le x < 3$

14. Let $F(x) = \displaystyle\int_0^x e^{-t^2}\,dt$. The first four non-zero terms of the Maclaurin series for $F(x)$ are:

(a) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$ (b) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!}$ (c) $x - \frac{x^3}{3} + \frac{x^5}{5\cdot 2!} - \frac{x^7}{7\cdot 3!}$

(d) $x - \frac{x^3}{3} - \frac{x^5}{5\cdot 2!} - \frac{x^7}{7\cdot 3!}$ (e) $1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!}$

15. Use an appropriate series to calculate $\displaystyle\int_0^{1/2} \frac{1}{1+x^4}\,dx$ to three decimal places.

(a) 0.491 (b) 0.492 (c) 0.493 (d) 0.494 (e) 0.495

16. Which of the following is equal to $\ln x - \ln 3$ for x near 3?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{3^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(x-3)^n}{n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(x-3)^n}{n3^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-3)^n}{3^n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-3)^n}{n3^n}$