Final Exam - April 29, 2005
Math 104

Name:

Student ID Number:

Instructor:

Teaching Assistant:

Section Number: **Recitation Day/Time:**

There are twenty multiple choice questions on this examination. Show your work in the space provided, and then carefully transfer your answers to this sheet. **Please write legibly.** No calculators, books, or notes may be used except for one two-sided 8.5"x11" sheet of notes. **Good luck!**

Question	Answer	Points
1		/5
2		/5
3		/5
4		/5
5		/5
6		/5
7		/5
8		/5
9		/5
10		/5
11		/5
12		/5
13		/5
14		/5
15		/5
16		/5
17		/5
18		/5
19		/5
20		/5
TOTAL		/100

1. Find the **area of the surface** generated by revolving the curve $y = \frac{x^3}{3}$, $0 \le x \le 1$ about the **x-axis**.

   (A) 1 (B) $\frac{2}{3}$ (C) $\frac{1}{9}$ (D) $\frac{1}{12}$

   (E) $\frac{4\sqrt{2}\pi}{3}$ (F) $\frac{2\sqrt{2}\pi}{9}$ (G) $\frac{2\sqrt{2}\pi}{12}$

   (H) $\frac{2\pi}{3}\left(2\sqrt{2}-1\right)$ (I) $\frac{\pi}{9}\left(2\sqrt{2}-1\right)$ (J) $\frac{\pi}{12}\left(2\sqrt{2}-1\right)$

2. Find the **area** of the region bounded by $y = \frac{2}{x}$ and $y = -x + 3$.

   (A) 0 (B) $\frac{3\pi}{2}$ (C) $\sqrt{3}\pi$ (D) $\sqrt{6}\pi$

   (E) $\frac{3}{2} - \ln 4$ (F) $\sqrt{3} - \ln 4$ (G) $\sqrt{6} - \ln 4$

   (H) $\frac{3}{2} - \ln\sqrt{2}$ $\sqrt{3} - \ln\sqrt{2}$ $\sqrt{6} - \ln\sqrt{2}$

3. Find the **volume** of the solid obtained by revolving the region bounded by the x-axis, the curve $y = \ln x$, and the line $x = e$ about the **y-axis**.

(A) π (B) $\pi\left(e^2+1\right)$ (C) $\pi\left(e^2+4\right)$ (D) $\pi^2\left(e^2+1\right)$ (E) $\pi^2\left(e^2+4\right)$

(F) $\frac{\pi}{2}$ (G) $\frac{\pi}{2}\left(e^2+1\right)$ (H) $\frac{\pi}{2}\left(e^2+4\right)$ (I) $\frac{\pi^2}{2}\left(e^2+1\right)$ (J) $\frac{\pi^2}{2}\left(e^2+4\right)$

4. Find the **volume** of the solid obtained by revolving the region bounded by the x-axis, the curve $y = \sin x$, $0 \le x \le \pi$ about the **y-axis**.

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) 2

(E) π (F) $\frac{\pi}{2}$ (G) 2π

(H) π^2 (I) $\frac{\pi^2}{2}$ (J) $2\pi^2$

5. Find a curve through the origin whose **length** from a to b is given by the following integral.

$$L = \int_a^b \sqrt{1 + \left(\frac{1}{1+x^2}\right)^2}\, dx$$

(A) $\sec^{-1} x$ (B) $\tan^{-1} x$ (C) $\sin^{-1} x$ (D) $\ln\left(x^2+1\right)$ (E) $\frac{-1}{x^2+1}$

(F) $\sec^{-1} x - 1$ (G) $\tan^{-1} x - 1$ (H) $\sin^{-1} x - 1$ (I) $\ln\left(x^2+1\right) - 1$ (J) $\frac{-1}{x^2+1}$

6. Solve the following initial value problem.

$$\frac{dy}{dx} + \frac{3}{x}y = x^2, \qquad y(1) = 1$$

(A) $y = x$ (B) $y = \frac{1}{2}x^3$ (C)$y = \frac{1}{3}x^3$ (D) $y = \frac{1}{6}x^3$

(E) $y = \frac{1}{2}x^3 + \frac{1}{2}x^{-3}$ (F) $y = \frac{1}{3}x^3 + \frac{2}{3}x^{-3}$ (G) $y = \frac{1}{6}x^3 + \frac{5}{6}x^{-3}$

(H) $y = \frac{1}{2}x^3 - \frac{1}{2}x^{-3}$ (I) $y = \frac{1}{3}x^3 - \frac{2}{3}x^{-3}$ (J) $y = \frac{1}{6}x^3 - \frac{5}{6}x^{-3}$

7. The amount $y(t)$ of alcohol in the bloodstream declines at a rate proportional to itself. This rate varies from one person to another. If it takes two hours for a person's blood-alcohol level to drop from .10 to .08, what will the blood-alcohol level be after an additional 2 hours?

(A) 0 (B) $\frac{1}{15} \approx .066$ (C) $\frac{2}{33} \approx .061$ (D) $\frac{3}{50} = .060$ (E) $\frac{4}{63} \approx .063$
(F) $\frac{5}{72} \approx .070$ (G) $\frac{6}{77} \approx .078$ (H) $\frac{7}{120} \approx .058$ (I) $\frac{8}{125} = .064$ (J) $\frac{9}{160} \approx .056$

8. Evaluate the following limit.

$$\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$$

(A) 1 (B) e (C) e^2 (D) $\sqrt{e}$ (E) $e^2 - \sqrt{e}$
(F) $\frac{1}{e}$ (G) $\frac{1}{e^2}$ (H) $\frac{1}{\sqrt{e}}$ (I) $\frac{1}{e^2-\sqrt{e}}$ (J) The limit does not exist.

9. Evaluate the following definite integral.

$$\int_0^{\pi} e^x \sin x \, dx$$

(A) 1 (B) $\frac{1}{2}$ (C) π (D) $\frac{\pi}{2}$

(E) $e^{\pi}+1$ (F) $e^{2\pi}+1$ (G) $e^{2\pi}+e^{\pi}+1$

(H) $\frac{e^{\pi}+1}{2}$ (I) $\frac{e^{2\pi}+1}{2}$ (J) $\frac{e^{2\pi}+e^{\pi}+1}{2}$

10. Evaluate the following definite integral.

$$\int_2^{9} \frac{x+4}{(x+6)(x-1)} \, dx$$

(A) $\frac{2}{7}\ln 5$ (B) $\frac{2}{7}\ln 5+\frac{3}{7}\ln 15$ (C) $\frac{2}{7}\ln 8+\frac{3}{7}\ln 15$ (D) $\frac{2}{7}\ln 5+\frac{3}{7}\ln 12$ (E) $\frac{2}{7}\ln 8+\frac{3}{7}\ln 12$

(F) $\frac{3}{7}\ln 5+\frac{2}{7}\ln 15$ (G) $\frac{3}{7}\ln 8+\frac{2}{7}\ln 15$ (H) $\frac{3}{7}\ln 5+\frac{2}{7}\ln 12$ (I) $\frac{3}{7}\ln 8+\frac{2}{7}\ln 12$ (J) ∞

11. Evaluate the following definite integral.

$$\int_0^{\sqrt{5}} \frac{x^3}{\sqrt{x^2+4}}\,dx$$

(A) $\frac{7}{3}$ (B) $\frac{5}{2}$ (C) $\sqrt{5}$ (D) $\frac{7\sqrt{5}}{3}$ (E) $\frac{5\sqrt{5}}{2}$

(F) $\frac{7}{3}\tan^{-1}\sqrt{5}$ (G) $\frac{5}{2}\tan^{-1}\sqrt{5}$ (H) $\frac{7\sqrt{5}}{3}\tan^{-1}\sqrt{5}$ (I) $\frac{5\sqrt{5}}{2}\tan^{-1}\sqrt{5}$ (J) ∞

12. Evaluate the following definite integral.

$$\int_0^{2} \frac{1}{(x-1)^2}\,dx$$

(A) 0 (B) 2 (C) -2 (D) $\ln 2$ (E) $-\ln 2$

(F) $2+\ln 2$ (G) $2-\ln 2$ (H) $-2+\ln 2$ (I) $-2-\ln 2$ (J) ∞

13. Find the limit of the following **sequence**.

$$(\sin \pi n)^n$$

(A) 0 (B) 1 (C) π (D) π^2 (E) $\sqrt{\pi}$ (F) e (G) e^2 (H) $\sqrt{e}$ (I) $\pi + e$
(J) The sequence diverges.

14. The limit of the **sequence**

$$a_n = \frac{n(\sin(n) + 3)}{n^2 + 1}$$

is:

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) -1 (G) -2 (H) -3 (I) -4
(J) The sequence is divergent.

15. Consider the following four series:

$$\text{I. } \sum_{n=1}^{\infty} \frac{1}{n^2} \qquad \text{II. } \sum_{n=1}^{\infty} 2^n \qquad \text{III. } \sum_{n=1}^{\infty} \frac{1}{n^{1/2}} \qquad \text{IV. } \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$$

Which of the following statements is true?

(A) I, II, III, and IV all converge.

(B) I and II converge, but III and IV diverge.

(C) I and III converge, but II and IV diverge.

(D) I and IV converge, but II and III diverge.

(E) I converges, but II, III, and IV diverge.

(F) I diverges, but II, III, and IV converge.

(G) I and IV diverge, but II and III converge.

(H) I and III diverge, but II and IV converge.

(I) I and II diverge, but III and IV converge.

(J) I, II, III, and IV all diverge.

16. Consider the following three series.

$$\text{I. } \sum_{n=1}^{\infty} \frac{(-1)^n(n^3+7)}{n^5+3n^4} \qquad \text{II. } \sum_{n=1}^{\infty} \frac{(-1)^n}{n+1} \qquad \text{III. } \sum_{n=1}^{\infty} \frac{(-1)^n(n^2+7)}{3n^2+5n}$$

Which of the following statements is true?

(A) I, II, and III all converge absolutely.

(B) I converges absolutely, II diverges, and III converges conditionally.

(C) I converges absolutely, II converges conditionally, and III diverges.

(D) I, II, and III converge conditionally.

(E) I converges conditionally, and II and III converge absolutely.

(F) I converges conditionally, and II and III diverge.

(G) I, II, and III all diverge.

(H) I diverges, and II and III converge absolutely.

(I) I diverges, and II converges conditionally, and III converges absolutely.

(J) I and II diverge, and III converges conditionally.

17. The interval of convergence of the series

$$\sum_{n=2}^{\infty} \frac{(3x-2)^n}{\ln(n)}$$

is:

(A) 0 (B) $[2,3]$ (C) $[2,3)$ (D) $(2,3]$ (E) $(2,3)$

(F) $[\frac{1}{3},1]$ (G) $[\frac{1}{3},1)$ (H) $(\frac{1}{3},1]$ (I) $(\frac{1}{3},1)$ (J) $(-\infty,\infty)$

18. The coefficient of $(x-3)^5$ in the Taylor polynomial of order 7 of

$$x\ln(x) - 3\ln(x)$$

centered at $a = 3$ is:

(A) $\frac{1}{324}$ (B) $\frac{1}{42}$ (C) 0 (D) $-\frac{1}{42}$ (E) $-\frac{1}{324}$

19. Find the sum of the series

$$\sum_{n=3}^{\infty} \ln\left(\frac{n}{n+1}\right)^3.$$

(A) 0 (B) $3\ln(2)$ (C) $3\ln(3)$ (D) $3\ln(4)$ (E) The series is divergent.

20. If you use the Taylor polynomial of order 9 of $\sin(x^2)$ to approximate

$$\int_0^1 \sin(x^2)\,dx$$

you get:

(A) $\frac{1}{6}$ (B) $\frac{5}{6}$ (C) $\frac{8}{9}$ (D) $\frac{13}{42}$ (E) $\frac{951}{3080}$