University of Pennsylvania
Mathematics Department
Mathematics 104/Final Examination
Fall 2004

Name: _____ Penn ID #: _____

Professor/Section (check one): ❑ Braun/001 ❑ Leidy/002 ❑ Bleher/003 ❑ Bleher/004
❑ Matthews/005 ❑ Stovall/006 ❑ Stovall/007 ❑ Crotty/008

Your TA: _____

Instructions:

1. **DO NOT DETACH THIS SHEET FROM YOUR TEST PAPER.**
2. NO CALCULATORS, NO COMPUTERS, NO OTHER AIDS MAY BE USED ON THIS EXAM. A SINGLE 8.5 × 11 SQUARE INCH NOTE SHEET *IS* PERMITTED.
3. Cell phones and other electronic devices are not to be brought to the testing room.
4. Write only your answer for each question in the appropriate space below. Show all your work in the space provided in this test booklet.

Multiple Choice (5 points each)	**Multiple Choice (5 points each)**	**Multiple Choice (5 points each)**
1.	6.	11.
2.	7.	12.
3.	8.	13.
4.	9.	14.
5.	10.	

15.(I) **(3 pts)**	15.(II) **(4 pts)**	15.(III) **(3 pts)**

Free Response:

1. (a) **(2 points)** Standard Form: _____

   (b) **(3 points)** Integrating Factor: _____

   (c) **(5 points)** Solution: _____

2. (a) **(2 points)** Center: _____ (b) **(4 points)** Radius of convergence: _____

   (c) **(4 points)** Interval of convergence: _____

Part 1: Multiple Choice

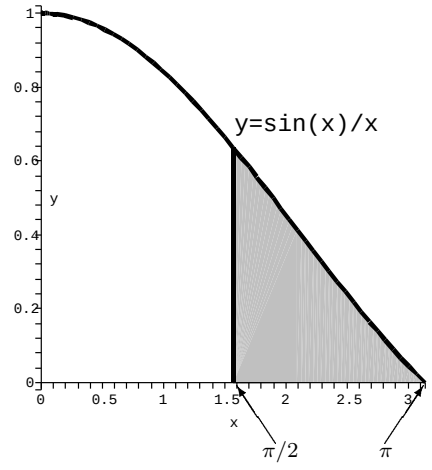
Work each problem in the space provided. Write the letter corresponding to your answer on your answer sheet. **For the multiple choice questions, no partial credit will be given.**

1. Evaluate $\int_1^e x^2 \ln x \, dx$.

   a) 0 b) 1 c) $\ln 2 - 1$ d) $\frac{2}{9}e^3 + \frac{1}{9}$ e) $\frac{2}{9}e^2 + \frac{1}{3}$ f) $\frac{1}{3}e^3 - 1$

2. Find the volume of the solid obtained by revolving the region below $y = \dfrac{\sin(x)}{x}$ and above the x-axis from $x = \pi/2$ to $x = \pi$ about the y-axis.

   a) 0 b) $\pi/3$ c) $\pi/2$ d) π e) $3\pi/2$ f) 2π



3. Find the length of the curve $y = \sqrt{1 - x^2}$ from $x = 0$ to $x = 1/2$.

a) 1 b) $\sqrt{3}/2$ c) $\sqrt{2} - 1$ d) $\pi/6$ e) $\pi/3$ f) $\pi/2$

4. If $y(x)$ satisfies the differential equation $\dfrac{dy}{dx} = e^{2x-y}$ and $y(0) = 1$, then $y(1/2)$ is

a) $\ln\left(\dfrac{3}{2}e - \dfrac{1}{2}\right)$ b) $1 - \dfrac{1}{\ln 2}$ c) $\dfrac{1}{2}(e + 1)$ d) e e) 2 f) 0

5. Evaluate $\displaystyle\int_2^3 \frac{1}{x^2(x-1)}\,dx$.

a) 1 b) $\frac{9}{4}$ c) $\ln 3$ d) $4\ln 2 - 1$ e) $4\ln 2 - 3\ln 3$ f) $2\ln 2 - \ln 3 - \frac{1}{6}$

6. If $f(x) = x^{\cos x}$, then $f'(\pi/2)$ is

a) 1 b) 0 c) $-\ln(\pi/2)$ d) $\frac{\pi}{2}\ln 2$ e) $-e^{\pi/2}$ f) undefined

7. Evaluate $\int_0^2 \frac{1}{(x-1)^2}\,dx$.

a) -2 b) 2 c) $-\frac{3}{5}$ d) $\frac{3}{5}$ e) 0 f) divergent

8. Evaluate $\int_0^4 \frac{dx}{(9+x^2)^{3/2}}$.

a) 1 b) $\frac{4}{45}$ c) $\frac{1}{11}$ d) $\frac{3}{10}$ e) $\ln 2$ f) $3\ln 5 - 1$

9. The limit of the sequence $\{a_n\} = \left\{\dfrac{n \ln n}{n^2+5}\right\}$ is

a) 0 b) $\dfrac{2}{5}$ c) $\dfrac{1}{2}$ d) 1 e) $\ln 2$ f) $\{a_n\}$ is divergent

10. Determine the convergence behavior of the following two alternating series:

(I) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n \ln n}$ (II) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{4^{n+1}}$

a) (I) converges absolutely, (II) converges conditionally
b) (I), (II) converge absolutely
c) (I) converges conditionally, (II) converges absolutely
d) (I), (II) converge conditionally
e) (I) diverges, (II) converges conditionally
f) (I) diverges, (II) converges absolutely

11. The series $\sum_{n=1}^{\infty} \frac{n+\sqrt{n}}{n^2+7}$

a) converges by comparison to $\sum_{n=1}^{\infty} \frac{1}{n}$ b) diverges by comparison to $\sum_{n=1}^{\infty} \frac{1}{n}$

c) converges by comparison to $\sum_{n=1}^{\infty} \frac{1}{n^2}$ d) diverges by comparison to $\sum_{n=1}^{\infty} \frac{1}{n^2}$

e) converges by the n^{th} root test f) diverges by the n^{th} root test

12. In the Taylor series generated by $f(x) = x^{1/3}$ and centered at $a = 1$, the coefficient of $(x-1)^2$ is

a) $-\frac{1}{9}$ b) $\frac{1}{9}$ c) $-\frac{2}{9}$ d) $\frac{2}{9}$ e) $-\frac{1}{3}$ f) $\frac{1}{3}$

13. In the Maclaurin series of $\int \frac{e^{x^2}-1}{x}\,dx$, the coefficient of x^6 is

a) $\frac{1}{4}$ b) $\frac{1}{6}$ c) $\frac{1}{8}$ d) $\frac{1}{18}$ e) $\frac{1}{36}$ f) $\frac{1}{48}$

14. Let $f(x) = x^2 \ln(1+x)$. To the nearest 0.00001, the value of $f(0.1)$ is

a) 0.00090 b) 0.00092 c) 0.00094 d) 0.00095 e) 0.00098 f) 0.00100

15. Determine which of the following series converge and which diverge:

(I) (3 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n+1)!\,n!\,2^n}$

a) convergent b) divergent

(II) (4 points) $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$

a) convergent b) divergent

(III) (3 points) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln(n)}{\ln(n^2)}$

a) convergent b) divergent

Part 2: Free Response

NOTE: If an answer involves quantities such as $\ln 4$ or e^{-3}, leave your results in that form and do not attempt to evaluate your answer as a decimal number.

1. Consider the initial value problem

$$x^4 \frac{dy}{dx} + 3x^3 y = x\, e^x, \qquad x > 0, \qquad y(1) = -1\ .$$

(a) (2 points) Write the equation in standard form.

(b) (3 points) Compute the integrating factor for the standard form.

(c) (5 points) Solve the equation subject to the initial condition.

2. Consider the power series

$$\sum_{n=1}^{\infty} \frac{(x-4)^n}{6^n\, n^{3/2}} \ .$$

(a) (2 points) What is the center (i.e. the point of expansion) for this series?

(b) (4 points) What is the radius of convergence of this series?

(c) (4 points) What is the interval of convergence for this series?