1	2	3	4	4	5	6	7	8	9	10	11	12	13	14	15	**Total**

D o n o t w r i t e a b o v e t h i s l i n e !

NAME (print):

Math 104 / Fall 2011

F I N A L E X A M

Rules:

- One sheet of paper ($8\frac{1}{2}$ by 11 inch) both sides handwritten notes is permitted.
- No other written or printed materials are allowed.
- No electronic devices (cellular, calculator, iPad, etc.) are allowed.

Grading:

- Each problem is worth 10 points (partial credit possible).
- Do all 15(fiveteen) problems, **showing your work** and **circling** your answers.
- No credit will be given for just guessing and not showing the work leading to the answer.

Instructions:

- Fill out the information requested below, and at the top of every page of this exam.
- Check that your exam booklet contains cover page + eight pages (15 problems).

Signature:

Class:

Recitation (#, day & time):

NAME:

1. The value of the integral $\displaystyle\int_{-1}^{1} \left(\sqrt[3]{x} + \frac{1}{1+x^2} + \frac{1}{2-x} \right) dx$ is:

(A) $1 + \frac{\pi}{2}$ (B) $\frac{47}{10}$ (C) $\frac{\pi}{2} + \ln 3$ (D) $\ln 3 + 3$ (E) $1 + 2\pi$ (F) 0 (G) $\frac{\pi}{2} - \ln 3$ (H) 1

2. Find the length of the arc of the curve defined by $y = \frac{2}{3}\sqrt{x^3}$ for $0 \leq x \leq 3$.

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) 4 (D) $5 \ln 3$ (E) $\frac{14}{3}$ (F) $\frac{1}{4}$ (G) $\frac{e}{8}$ (H) $\frac{\ln 3}{2}$

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3. Find the volume obtained by rotating the region between the graph of $y = \frac{1}{2}\sin^2(x^2)$ and the x-axis for $0 \leq x \leq \sqrt{\pi}$ about the y-axis.

(A) $\frac{\pi}{2}$ (B) $\frac{\pi^2}{4}$ (C) $\frac{5}{4}$ (D) $\frac{3\pi^2}{4}$ (E) $\frac{1}{2}$ (F) $\frac{1}{4}$ (G) $\frac{\pi}{8}$ (H) $\frac{\pi^2}{8}$

4. Evaluate $\displaystyle\int_1^{e^3} \frac{\ln x}{\sqrt[3]{x^2}}\, dx$.

(A) $3e - 9$ (B) $3e^2 - 9$ (C) $9e^2 - 3$ (D) $3e^2$ (E) $9e^2$ (F) 9 (G) $9e - 3$ (H) $3e$

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5. Find the area bounded by the x-axis and the graph of $y = xe^{-2x}$ for $0 \leq x < \infty$.

(A) 1 (B) 2 (C) $e - 2$ (D) $\frac{1}{4}$ (E) $\frac{1}{2}$ (F) $\frac{1}{e}$ (G) $\frac{1}{2e}$ (H) $\frac{1}{4e}$

6. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(5x-3)^n}{n^2}$.

(A) $(-1, 1)$ (B) $[-1, 1]$ (C) $[1, \frac{4}{5})$ (D) $[-\frac{4}{5}, \frac{4}{5}]$ (E) $[-\frac{4}{5}, \frac{4}{5})$ (F) $[\frac{2}{5}, \frac{4}{5}]$ (G) $[0, 1]$ (H) $\{0\}$

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7. Let $f(x) = e^{-x^2}$. Then $f^{(10)}(0)$ is

(A) $-\frac{1}{120}$ (B) $\frac{1}{10!}$ (C) $\frac{10}{5!}$ (D) $-\frac{10!}{5!}$ (E) $\frac{3}{10}$ (F) $\frac{1}{100}$ (G) 1 (H) 0

8. The region bounded by $y = \dfrac{x}{\sqrt[4]{(x^2+3)^5}}$, the x-axis, and $0 \leq x \leq 1$, is rotated about the x-axis. The volume of the resulting solid is equal to:

(A) $\frac{\pi}{6}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{e}{2}$ (D) $\frac{\pi}{4}$ (E) $\frac{e}{2}$ (F) $\sec 2$ (G) $\frac{1}{2}$ (H) $\frac{\pi}{72}$

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9. Which of the following is the best approximation of $\ln(\frac{11}{10})$?

(A) 0 (B) $\frac{1}{10}$ (C) $\frac{5}{100}$ (D) $\frac{9}{100}$ (E) $\frac{95}{1000}$ (F) $\frac{99}{1000}$ (G) $\frac{109}{1000}$ (H) $\frac{155}{1000}$

10. Consider the function $f(x) = \frac{1}{x}\, e^{-x^2} \sin 2x$ for $x \neq 0$ and $f(0) = 2$. The order three Taylor polynomial $a_0 + a_1 x + a_2 x^2 + a_3 x^3$ of $f(x)$ about $x = 0$ is:

(A) $2 - \frac{10}{3}x^2$ (B) $2x - \frac{4}{3}x^3$ (C) $2 - \frac{4}{3}x^2$ (D) $2 - x^2$
(E) $x - \frac{1}{3}x^3$ (F) $1 + x - x^3$ (G) $-2 + x + \frac{10}{3}x^2$ (H) $2 - x + x^2$

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11. Let $y(x)$ be the solution to the initial-value problem $x\dfrac{dy}{dx} - 2y = x^3$ and $y(1) = 0$. What is $y(3)$?

(A) 1 (B) 3 (C) 6 (D) 9 (E) 12 (F) 15 (G) 18 (H) 27

12. A random variable has as probability density function $p(x) = 2(x+1)^{-3}$ for $x \geq 0$ and $p(x) = 0$ else. What is the mean of the random variable?

(A) $\sqrt{2}$ (B) $\frac{3}{2}$ (C) 1 (D) $2\sqrt{2}$ (E) 2 (F) 4 (G) 0 (H) $\frac{11}{2}$

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13 Evaluate the integral $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\tan t}{\ln(\cos t)}\, dt.$

(A) $\frac{1}{4e}$ (B) $\frac{1}{2e} - \frac{1}{e}$ (C) $\ln 2$ (D) $2e + \frac{2}{e}$ (E) $\frac{2}{e}$ (F) $-\ln 2$ (G) $\ln 3 - 1$ (H) $\frac{4}{e}$

[**Hint**: $\tan t = \frac{\sin t}{\cos t}$, etc.]

14 Which of the following series converge?

(I) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$ (II) $\displaystyle\sum_{n=2}^{\infty} \frac{n^3}{\ln n}$ (III) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$ (IV) $\displaystyle\sum_{n=1}^{\infty} e^{1/n}$

(A) I & II (B) I & III (C) I & IV (D) II & III
(E) II & IV (F) III & IV (G) all four of them (H) none of them

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15. The values of $p \geq 0$ for which the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges are precisely:

(A) $p > 1$ (B) $p > 0$ (C) $p \geq 1$ (D) $p \leq 1$ (E) $p < 1$ (F) $p > \frac{1}{2}$ (G) $p > 2$ (H) none.