University of Pennsylvania | **Jankowski, Rimmer, Jauregui, Sazdanovic**

Math 104 Spring 2012 Final Exam | **Answers provided on the last page**

1. Determine the limit of the sequence

$$a_n = n\left(2^{1/n} - 1\right)$$

A) $\ln 2$ B) $\frac{1}{e}$ C) e^2 D) 0 E) 1 F) ∞

2. Determine whether the following series are convergent or divergent. For full credit be sure to explain your reasoning and tell what test was used.

$$(\mathrm{I})\sum_{n=1}^{\infty}\frac{e^{1/n}}{n^2} \qquad (\mathrm{II})\sum_{n=1}^{\infty}\left(\frac{8n}{5+7n}\right)^n \qquad (\mathrm{III})\sum_{n=1}^{\infty}\frac{\arctan(n)}{\sqrt{n^7}}$$

	(I)	(II)	(III)
A)	convergent	convergent	convergent
B)	convergent	convergent	divergent
C)	convergent	divergent	convergent
D)	convergent	divergent	divergent
E)	divergent	convergent	convergent
F)	divergent	convergent	divergent
G)	divergent	divergent	convergent
H)	divergent	divergent	divergent

3. Find the interval of convergence of

$$\sum_{n=1}^{\infty}\frac{(3x-2)^n}{3^n n\sqrt{n}}$$

A) $\left(\frac{-1}{3},\frac{5}{3}\right]$ B) $\left[\frac{-1}{3},\frac{5}{3}\right)$ C) $\left(\frac{-1}{3},\frac{5}{3}\right)$

D) $\left[\frac{-1}{3},\frac{5}{3}\right]$ E) $\left(\frac{1}{3},1\right]$ F) $\left[\frac{1}{3},1\right)$

4. Which of the following is an approximation to $\sin(2)$ with error less than $\frac{1}{10}$?

Hint: Use an appropriate series centered at 0.

A) 2 B) $\frac{3}{2}$ C) 1 D) $\frac{14}{15}$ E) $\frac{2}{3}$ F) $\frac{-23}{45}$

5. Find the fifth order Taylor polynomial about $x=0$ for

$$f(x)=\int_0^x e^{t^2}\,dt$$

A) $1+x^2+\frac{x^4}{2}$ B) $x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{120}$ C) $1+x+\frac{x^3}{6}+\frac{x^5}{10}$

D) $x+\frac{x^3}{3}+\frac{x^5}{10}$ E) $x+\frac{x^4}{4}$ F) $x+\frac{x^3}{3}+\frac{x^5}{5}$

6. The distance $y(t)$ traveled by some object satisfies the equation

$$\sqrt{t}\,\frac{dy}{dt}=e^{\sqrt{t}-y}, \quad t>\tfrac{1}{2}.$$

If $y(1)=1$, what is $y(4)$?

A) $\ln\left(3e^3-e\right)$ B) $\ln\left(4e-e^2\right)$ C) $\ln\left(2e^2\right)$

D) $\ln\left(3e-1\right)$ E) $\ln\left(2e^2-e\right)$ F) 4

7. Find the solution to the differential equation

$$xy'=\frac{2y}{x-2}+x^2-2x, \quad x>2$$

satisfying $y(4)=4$.

A) $y=\frac{x^3-3x^2}{4}$ B) $y=\frac{x^2-2x}{2}$ C) $y=e^{x^2}-e^{4x}+4$

D) $y=\frac{\ln(x)-x}{2}$ E) $y=\frac{x^2-x}{3}$ F) $y=\frac{x^3+x}{17}$

8. Evaluate the integral

$$\int 4\sin^2(3x)\cos^2(3x)\,dx.$$

A) $\frac{1}{2}x-\frac{1}{24}\sin(12x)+C$ B) $\frac{1}{2}x-\frac{1}{8}\sin(4x)+C$ C) $\frac{1}{3}\cos^3(3x)+C$

D) $-\frac{1}{3}\sin^3(3x)+C$ E) $x-\frac{1}{6}\sin^2(6x)+C$ F) $x-\frac{1}{6}\cos^3(6x)+C$

9. The integral

$$\int \frac{9dx}{x^2(x+3)}$$

is equal to

A) $-\frac{9}{x}\ln|x+3|+C$ B) $-\ln|x|+\ln|x+3|-\frac{3}{x}+C$ C) $9\ln|x^2(x+3)|+C$

D) $-9\ln|x|+9\ln|x+3|-\frac{3}{x}+C$ E) $-\frac{9}{x}+\frac{1}{\sqrt{3}}\arctan\left(\frac{\sqrt{x}}{\sqrt{3}}\right)+C$ F) $\ln|x|-\frac{1}{x}+\ln|x+3|+C$

10. The integral

$$\int_0^1 \frac{dx}{(x^2+4)^{3/2}}$$

Is equal to

A) $\frac{1}{2}\arctan\left(\frac{1}{2}\right)$ B) $-\dfrac{2}{\sqrt{5}}+1$ C) $\frac{1}{4}\ln\left(\frac{\sqrt{5}}{2}\right)$

D) $\dfrac{1}{2\sqrt{5}}-\dfrac{1}{4}$ E) $\dfrac{1}{4\sqrt{5}}$ F) diverges

11. The integral

$$\int_1^{\infty} x^2 e^{-2x} dx$$

is equal to

A) 1 B) $-e^{-2}$ C) $\frac{5}{4}e^{-2}$

D) $\dfrac{1}{4}$ E) $\frac{1}{4}e^{-2}$ F) diverges

12. What is the area of the surface obtained by rotating the part of the curve $y=\sqrt{4-x^2}$ from $x=0$ to $x=1$ around the $x-$axis?

A) 4π B) 2π C) π D) $\sqrt{2}\pi$ E) 3π F) 8π

13. Find the volume of the solid generated by revolving the region bounded by the graphs of $y=x^2$ and $y=1$ around the $x-$axis.

A) $\dfrac{8}{5}\pi$ B) $\dfrac{1}{5}\pi^2$ C) $\dfrac{4}{5}\pi$ D) 2π E) $\dfrac{14}{15}\pi$ F) $\dfrac{16}{5}\pi$

14. Which of the following integrals can be used to compute the volume of the solid generated by revolving the region beneath the graph of $y=1-x^2$, to the right of the $y-$axis and above the $x-$axis about the line $x=3$?

A) $\int_0^3 \pi\left(1-x^2\right)^2 dx$ B) $\int_0^1 2\pi(3)\left(x^2-1\right)dx$ C) $\int_0^1 2\pi x\left(1-x^2\right)dx$

D) $\int_0^1 2\pi(3-x)\left(1-x^2\right)dx$ E) $\int_0^1 \pi(8+y)\,dy$ F) $\int_0^1 \pi(3-x)^2\left(x^2-1\right)dx$

15. The length of time spent waiting in line at the security check at the Philadelphia International Airport is modeled by an exponential density function with a mean of half an hour. With time measured in hours, the exponential density function would be

$$f(t) = \begin{cases} 2e^{-2t} & t \geq 0 \\ 0 & t < 0 \end{cases}.$$

What is the probability that it will take longer than two hours for a passenger to make it through the security check?

A) 0 B) 1 C) $\frac{1}{30}$ D) $\frac{1}{e^2}$ E) $\frac{1}{e^4}$ F) $\frac{1}{e^8}$

ANSWERS:

1. (A) (B) (C) (D) (E) (F)
2. (A) (B) (C) (D) (E) (F) (G) (H)
3. (A) (B) (C) (D) (E) (F)
4. (A) (B) (C) (D) (E) (F)
5. (A) (B) (C) (D) (E) (F)
6. (A) (B) (C) (D) (E) (F)
7. (A) (B) (C) (D) (E) (F)
8. (A) (B) (C) (D) (E) (F)
9. (A) (B) (C) (D) (E) (F)
10. (A) (B) (C) (D) (E) (F)
11. (A) (B) (C) (D) (E) (F)
12. (A) (B) (C) (D) (E) (F)
13. (A) (B) (C) (D) (E) (F)
14. (A) (B) (C) (D) (E) (F)
15. (A) (B) (C) (D) (E) (F)