MATH 104 MAKEUP FINAL EXAM

—for the Fall 2012 Term—

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1. Compute the improper integral: $\displaystyle\int_{-1}^{0} \Big(\frac{4}{x^2+1} + \sin(2\pi x) - x^{-\frac{1}{3}}\Big)dx$

(a) $1+2\pi$ (b) 0 (c) divergent (d)* $\frac{3}{2}+\pi$ (e) $2\pi - \ln 3$ (f) $\ln 4 + \frac{3}{4}$

2. The area of the region bounded by $y = \cos(2\pi x)$, the x-axis, and the vertical lines $x = 0$ and $x = \frac{1}{2}$ is:

a) $\frac{2}{\pi}$ (b)* $\frac{1}{\pi}$ (c) $\frac{3}{2}$ (d) 2 (e) $\frac{5}{3}$ (f) 4

3. The region of the xy-plane bounded by $y = e^{-x/2}$ and the x-axis for $0 \le x \le \ln(2)$ is rotated about the x-axis. The volume of the resulting solid of revolution is:

(a) $\frac{2}{3}\pi$ (b) $\frac{1}{3}\pi$ (c) $\frac{3}{2}$ (d) 2π (e) $\frac{5}{3}$ (f)* $\frac{1}{2}\pi$

4. The length of the arc of curve $y = \frac{1}{3}x^3 + \frac{1}{4x}$ for $1 \le x \le 2$ is:

(a)* $\frac{59}{24}$ (b) $\frac{1}{3}\pi(5^{\frac{3}{2}} - 1)$ (c) $\frac{19}{6}\pi$ (d) 2π (e) $\frac{1}{2}\ln 2$ (f) πe^2

[**Hint:** Eventually use the identity: $a^4 + \frac{1}{2} + \frac{1}{16a^4} = (a^2 + \frac{1}{4a^2})^2$, etc...]

5. The sequence $x_n = \dfrac{\sqrt{2n^{11}-1}}{1-3n^5\sqrt{n}}$ is:

(a) divergent to ∞ (b) divergent to $-\infty$ (c) unbounded (d)* convergent

6. The interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}(2x+1)^n$ is:

(a) $[-1,0]$ (b)* $(-1,0]$ (c) $x = 0$ (d) $(0,1)$ (e) diverges (f) $(0,1]$

7. Suppose $y = y(x)$ satisfies the differential equation $x\,y' \ln x = x^2 \ln x - y$ and the initial condition $y(e) = \frac{1}{4}e^2$. Then $y(e^{\frac{1}{2}})$ is:

(a)* 0 (b) π (c) $-\pi$ (d) $-\frac{1}{\pi}$ (e) $\frac{1}{\pi}$ (f) 1

8. The volume of the solid of revolution obtained by rotating the region bounded by $y = x^2\sin(x^2)$ and the x-axis for $0 \le x \le \sqrt{\pi}$ about the y-axis is:

a) $\frac{2}{3}\pi$ (b) $\frac{1}{2}\pi$ (c) $\frac{3}{2}$ (d) $2\pi(e-1)$ (e)* π^2 (f) $\pi - \frac{2\pi}{e}$

9. Which of the assertions below hold for the following series:

$$\text{I}: \quad \sum_{n=1}^{\infty} \frac{(-e)^{n-1}}{n^2\pi^n} \qquad \text{II}: \quad \sum_{n=1}^{\infty} \frac{n}{\sqrt{7n^3-6n}} \qquad \text{III}: \quad \sum_{n=0}^{\infty} \frac{2^n-4^n}{3^n+4^n}$$

(a) I, II, III are convergent (b) I, II, III are divergent (c)* only I converges

(d) only I and III converge (e) only I and III diverge (f) only III converges

10. Compute the definite integral $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin^3(x)}{\cos^4(x)}dx$.

(a) 0 (b)* $\frac{4}{3}$ (c) $\arccos\frac{1}{3}$ (d) $\frac{5}{24}$ (e) $\frac{4}{35}$ (f) $\frac{6}{\sqrt{6}}$

11. Evaluate the integral $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\tan t}{\ln(\cos t)}\,dt$.

(a) $\frac{1}{4e}$ (b) $\frac{1}{2e}-\frac{1}{e}$ (c) $\ln 2$ (d) $2e+\frac{2}{e}$ (e) $\frac{2}{e}$ (f)* $-\ln 2$

[**Hint**: $\tan t = \frac{\sin t}{\cos t}$, etc.]

12. Consider the probability density function $f(x)$ defined by $f(x) = \dfrac{kx}{(x+1)^4}$ for $x \geq 0$ and $f(x) = 0$ for $x < 0$. Then the value of k is:

(a)* 6 (b) 1 (c) $\frac{3}{2}$ (d) 2 (e) $\frac{5}{3}$ (f) 4

13. Which of the following numbers is closest to $\sqrt[10]{\frac{3}{2}}$?

(a) 1 (b)* $\frac{104}{100}$ (c) $\frac{108}{100}$ (d) $\frac{112}{100}$ (e) $\frac{116}{100}$ (f) $\frac{12}{10}$

[**Hint:** $\sqrt[10]{\frac{3}{2}} = \left(1+\frac{1}{2}\right)^{\frac{1}{10}}$, etc...]

14. Consider the function $f(x) = \left(\cos(x^2) - 1 + \dfrac{x^4}{2}\right)/x^4$. Then $f^{(8)}(0)$ is:

(a) 24 (b) $\frac{1}{24}$ (c) $\frac{1}{6}$ (d) 8! (e)* -56 (f) 12

[**Hint:** Use the Maclaurin power series for $\cos(u)$, etc...]

15. For which values of α is the improper integral $\displaystyle\int_0^1 \frac{\ln(1+x)}{x^\alpha}\,dx$ convergent?

(a) all α (b) none (c) $\alpha = \frac{5}{2}$ only (d) $2 < \alpha$ (e)* $\alpha < 2$ (f) $\alpha = 5$ only

[**Hint:** One might use power series, etc...]