MATH 104 FINAL EXAM

—Fall 2012 Term—

1. Compute the integral: $\displaystyle\int_{-1}^{0} \Big(\frac{2}{x+2} + \cos(\pi x) - x^{\frac{1}{3}}\Big)dx$

(a) $1+\pi$ (b) 0 (c) divergent (d) $\pi + \frac{1}{4}$ (e) $2\pi - \ln 3$ (f)* $\ln 4 + \frac{3}{4}$

2. The area of the region bounded by $y = \sin(\pi x)$, the x-axis, and the vertical lines $x = -\frac{1}{2}$ and $x = \frac{1}{2}$ is:

a)* $\frac{2}{\pi}$ (b) 2π (c) $\frac{3}{2}$ (d) 2 (e) $\frac{5}{3}$ (f) 4

3. The region of the xy-plane bounded by $y = (x-1)^{\frac{1}{4}}$ and the x-axis for $1 \le x \le 2$ is rotated about the x-axis. The volume of the resulting solid of revolution is:

(a)* $\frac{2}{3}\pi$ (b) $\frac{1}{2}\pi$ (c) $\frac{3}{2}$ (d) 2π (e) $\frac{5}{3}$ (f) 4

4. The area of the surface obtained by rotating the arc of curve $y = \sqrt{x}$, $\frac{3}{4} \le x \le 2$, about the x-axis is:

(a) $\frac{1}{6}\pi(3 - 5^{\frac{3}{2}})$ (b) $\frac{1}{3}\pi(5^{\frac{3}{2}} - 1)$ (c)* $\frac{19}{6}\pi$ (d) 2π (e) $\frac{1}{2}\ln 2$ (f) πe^2

5. The sequence $x_n = \dfrac{2n\sqrt{n}}{1-3n^3}$ is:

(a) divergent to ∞ (b) divergent to $-\infty$ (c) unbounded (d)* convergent

6. The interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x-7)^n$ is:

(a) $[6,8]$ (b)* $(6,8]$ (c) $x = 5$ (d) $(6,8)$ (e) diverges (f) $(-1,1)$

7. Suppose $y = y(x)$ satisfies the differential equation $xy' = \cos x - y$ and the initial condition $y(\frac{\pi}{2}) = 0$. Then $y(\pi)$ is:

(a) 0 (b) π (c) $-\pi$ (d)* $-\frac{1}{\pi}$ (e) $\frac{1}{\pi}$ (f) 1

8. The volume of the solid of revolution obtained by rotating the region bounded by $y = x^2 e^{-x^2}$ and the x-axis for $0 \le x \le 1$ about the y-axis is:

a) $\frac{2}{3}\pi$ (b) $\frac{1}{2}\pi$ (c) $\frac{3}{2}$ (d) $2\pi(e-1)$ (e) $\frac{5}{3}\pi e$ (f)* $\pi - \frac{2\pi}{e}$

9. Which of the assertions below hold for the following series:

$$\text{I}: \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} \qquad \text{II}: \sum_{n=1}^{\infty} \frac{n}{\sqrt{7n^5-6n}} \qquad \text{III}: \sum_{n=0}^{\infty} \frac{2^n-5^n}{3^n+4^n}$$

(a) I, II, III are convergent (b) I, II, III are divergent (c) only I converges

(d)* only I and II converge (e) only I and III diverge (f) only III converges

10. Compute the definite integral $\int_0^{\pi} \sin^3(x)\cos^4(x)dx$.

(a) 0 (b) $\frac{4}{3}$ (c) $\arccos\frac{1}{3}$ (d) $\frac{1}{\sqrt{2}}$ (e)* $\frac{4}{35}$ (f) $\frac{6}{\sqrt{6}}$

11. Suppose that the region bounded by $y = 4\tan(x^2)$ and the x-axis for $0 \le x \le \frac{\sqrt{\pi}}{2}$ is a thin homogeneous density plate of area A. Then the x-coordinate of the center of mass of the plate is:

(a) $\frac{2}{A}\pi^2$ (b) $\frac{2}{A}\pi$ (c)* $\frac{1}{A}\ln 2$ (d) $\frac{3}{A}\sqrt{\pi}$ (e) 0 (f) $\frac{e\pi}{2}$

12. Consider the probability density function $f(x)$ defined by $f(x) = 2(x+1)^{-3}$ for $x \ge 0$ and $f(x) = 0$ for $x < 0$. Then the **mean** of the probability density function $f(x)$ is:

(a) 0 (b)* 1 (c) $\frac{3}{2}$ (d) 2 (e) $\frac{5}{3}$ (f) 4

13. Which of the following numbers is closest to $\sin(18^\circ)$?

(a) $\frac{316}{1000}$ (b) $\frac{313}{1000}$ (c)* $\frac{31}{100}$ (d) $\frac{307}{1000}$ (e) $\frac{304}{1000}$ (f) $\frac{301}{1000}$

[**Hint:** 18° in radias is $\frac{\pi}{10}$, etc...]

14. What is the coefficient of x^3 in the Maclaurin series of the function $f(x) = \dfrac{\sin x}{e^x}$?

(a) $-\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{2}{3}$ (e)* $\frac{1}{3}$ (f) $\frac{5}{6}$

[**Hint:** $\dfrac{1}{e^x} = e^{-x}$ and so on...]

15. For which values of α is the improper integral $\int_0^1 \dfrac{e^x-1}{x^\alpha}\,dx$ convergent?

(a) all α (b) none (c) $\alpha = \frac{5}{2}$ only (d) $2 < \alpha$ (e)* $\alpha < 2$ (f) $\alpha = 5$ only

[**Hint:** One might use power series, etc...]