Fall 2010 Math 104 Final

Name: Section number:

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Instructions

- No calculators please.
- One double sided 8.5" x 11" hand written sheet of notes is allowed.
- Answer choices not transferred to the front page will receive no credit.

Problem 1	A	B	C	D	E	F	G	H
Problem 2	A	B	C	D	E	F	G	H
Problem 3	A	B	C	D	E	F	G	H
Problem 4	A	B	C	D	E	F	G	H
Problem 5	A	B	C	D	E	F	G	H
Problem 6	A	B	C	D	E	F	G	H
Problem 7	A	B	C	D	E	F	G	H
Problem 8	A	B	C	D	E	F	G	H
Problem 9	A	B	C	D	E	F	G	H
Problem 10	A	B	C	D	E	F	G	H
Problem 11	A	B	C	D	E	F	G	H
Problem 12	A	B	C	D	E	F	G	H
Problem 13	A	B	C	D	E	F	G	H
Problem 14	A	B	C	D	E	F	G	H
Problem 15	A	B	C	D	E	F	G	H

Extra space

1. Find the area bounded by the y-axis, the graph of $y = e^x$ and the graph of $y = xe^x$.

(A) 1 (B) 2 (C) $e - 2$ (D) e (E) $e + 2$ (F) $2e$ (G) $e^2 - e$ (H) $3e - 2$

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2. Find the average value of the function $\sin^3(x)\cos^2(x)$ over the interval $[0, \pi]$.

(A) $\frac{4}{15\pi}$ (B) $\frac{2}{\pi}$ (C) $\frac{1}{8}$ (D) $\frac{4}{35\pi}$ (E) $-\frac{4}{15\pi}$ (F) $\frac{1}{2\pi}$ (G) $\frac{4}{15}$ (H) $\frac{8}{\pi}$

3. Find the volume of the solid obtained by rotating the region bounded by the x-axis, the line $y = 1$, the curve $y = \ln(x)$, and the line $x = 1/2$ about the y-axis.

(A) $\pi(e-2)$ (B) $2\pi\left(\frac{e^2}{4} - \frac{3}{4}\right)$ (C) $2\pi\left(\frac{e^2}{4} + \frac{3}{4}\right)$ (D) $\pi\left(\frac{1}{2}e^2 - \frac{3}{4}\right)$

(E) $\frac{\pi}{8}(4e^2 - 3 - 2\ln 2)$ (F) $\pi\left(e - \frac{3}{2}\right)$ (G) $\frac{e\pi}{2}$ (H) $\pi\left(\frac{3}{4} + \frac{e^2}{2} - e\right)$

4. Evaluate

$$\int \frac{dx}{x^2 - 2x - 3}$$

(A) $\dfrac{1}{\sqrt{3}} \arctan(\sqrt{3}x) + C$

(B) $-\dfrac{1}{x} - \dfrac{1}{2}\ln(x) - \dfrac{x}{3} + C$

(C) $-\dfrac{1}{8}\left(\dfrac{1}{x+1} + \dfrac{1}{x-3}\right) + C$

(D) $\dfrac{1}{2}\ln|x-1| + C$

(E) $\dfrac{2}{3}\ln|x-3| + \ln|x-1| + C$

(F) $\dfrac{1}{3}\ln\left|\dfrac{x-1}{x-3}\right| + C$

(G) $-\dfrac{1}{(x-3)^2} + \dfrac{1}{(x+1)^2} + C$

(H) $\dfrac{1}{4}\ln\left|\dfrac{x-3}{x+1}\right| + C$

5. Evaluate the integral: $\displaystyle\int_1^{e^2} \frac{\ln x}{\sqrt{x}}\, dx$

(A) $e - 1$ (B) $2e$ (C) e (D) $\frac{4}{3}e$

(E) 4 (F) $4 - 4\ln 2$ (G) $4 + 4\ln 2$ (H) $4\ln 2$

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6. Evaluate the integral

$$\int_0^{\pi/6} \frac{\sqrt{\sin(x)}}{\tan(x)} dx$$

(A) $\frac{1}{\sqrt{2}}$ (B) $-\sqrt{2}$ (C) $\sqrt{2}$ (D) $\sqrt{2}-2$

(E) $\frac{3}{\sqrt{2}}$ (F) $-\frac{1}{\sqrt{2}}$ (G) $\frac{\sqrt{3}}{\sqrt{2}}-2$ (H) The integral diverges.

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7. What is the arclength of the part of the curve $y = \frac{1}{12}e^x + 3e^{-x}$ for $\ln 2 \leq x \leq \ln 4$?

(A) $\frac{5}{12}$ (B) $\frac{1}{2}$ (C) $\frac{7}{12}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$ (F) $\frac{5}{6}$ (G) $\frac{11}{12}$ (H) 1

8. Find the solution to the initial-value problem

$$\frac{dy}{dx} = \frac{e^{-\sqrt{x}}}{y^2\sqrt{x}} \qquad y(0) = 3.$$

(A) $y = (33 - 6e^{-\sqrt{x}})^{1/3}$

(B) $y = (45 - 18e^{-\sqrt{x}})^{1/3}$

(C) $y = (9 + 18e^{-\sqrt{x}})^{1/3}$

(D) $y = (30 - 3e^{-\sqrt{x}})^{1/3}$

(E) $y = \dfrac{3}{(2 - e^{-\sqrt{x}})^{1/3}}$

(F) $y = \dfrac{9}{(45 - 18e^{-\sqrt{x}})^{1/3}}$

(G) $y = \dfrac{9}{(9 + 18e^{-\sqrt{x}})^{1/3}}$

(H) $y = \dfrac{3}{(9 - 8e^{-\sqrt{x}})^{1/3}}$

9. In a certain chemical reaction, the amount A of reactant decreases at a rate proportional to A^2. That is

$$\frac{dA}{dt} = -kA^2$$

for some positive constant k. If there are 4 grams of the reactant present at time $t = 0$, and there are 2 grams of it present at time $t = 10$ seconds, at what time will there be 1 gram of the reactant present?

(A) $t = 15$ seconds (B) $t = 20$ seconds (C) $t = 25$ seconds (D) $t = 30$ seconds

(E) $t = 45$ seconds (F) $t = 60$ seconds (G) $t = 90$ seconds (H) $t = 100$ seconds

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10. Which statement is true for

$$\text{(I)} \sum_{n=1}^{\infty} e^n \qquad \text{(II)} \sum_{n=1}^{\infty} \frac{1}{n^e} \qquad \text{(III)} \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} \qquad \text{(IV)} \sum_{n=1}^{\infty} \frac{n^2}{n^2+1}\,.$$

(A) All four series converge.
(B) None of the series converge.
(C) Only (I) and (II) converge.
(D) Only (I) and (III) converge.
(E) Only (I) and (IV) converge.
(F) Only (II) and (III) converge.
(G) Only (II) and (IV) converge.
(H) Only (III) and (IV) converge.

11. Find the value of b for which

$$\sum_{n=0}^{\infty} e^{nb} = 1 + e^b + e^{2b} + e^{3b} + \cdots = 9$$

(A) $e \ln\left(\frac{1}{9}\right)$ (B) $\ln\left(\frac{1}{9}\right)$ (C) $\frac{-1}{9}$ (D) $\frac{-e}{9}$

(E) $\ln\left(\frac{8}{9}\right)$ (F) $\ln\left(\frac{9}{10}\right)$ (G) $\ln\left(\frac{10}{9}\right)$ (H) $\ln(10)$

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12. Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{2^n n}$$

(A) $[-1, 3]$ (B) $[-1, 3)$ (C) $(-1, 3]$ (D) $(-1, 3)$

(E) $[1/2, 3/2]$ (F) $[1/2, 3/2)$ (G) $(1/2, 3/2]$ (H) $(1/2, 3/2)$

13. Compute the Maclaurin series (i.e., the Taylor series about 0) of

$$f(x) = x^2 + \arcsin(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \cdots$$

up to and including terms of order two. Then: evaluate $a_0^2 + a_1^2 + a_2^2$.

(Hint: $\dfrac{d}{dx} \arcsin(x) = \dfrac{1}{\sqrt{1-x^2}}$.)

(A) 0 (B) 1 (C) 2 (D) $\dfrac{9}{4}$ (E) 3 (F) $\dfrac{13}{4}$ (G) 4 (H) 5

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14. Which of the following approximates the integral

$$\int_0^1 \frac{-1+\cos t}{t^2}dt$$

to within 0.01?

(A) $\frac{1}{3600}$ (B) $-1-\sin(1)$ (C) $1-\cos(1)$ (D) $-\frac{1}{3}(1+\sin(1))$

(E) $-\frac{1}{3}$ (F) $-\frac{1}{2}$ (G) $\frac{1}{2}$ (H) $-\frac{35}{72}$

15. Compute the following improper integral:

$$\int_{x=0}^{\infty} \left(\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!} \right)^2 dx$$

(A) -1 (B) 0 (C) $\frac{1}{2}$ (D) 1

(E) 2 (F) π (G) 4 (H) The integral diverges