MATH 104, Fall 2009, Final Exam

NAME:

Student ID number:

Lecturer:

TA:

Recitation meeting time:

Problem	Score	Out of
1		6
2		6
3		6
4		6
5		5
6		5
7		5
8		5
9		6
10		5
11		7
12		6
13		8
14		8
15		8
16		8

Multiple choice questions

The worth of each question is one less than the number of possible answers, for example, if the choices are (a), (b), (c), (d), and (e), then a correct answer is worth 4 points. You do not need to justify your answer but you must SHOW YOUR WORK so we know you actually did it. An incorrect answer is worth -1. No calculators are allowed on this exam.

1. Find the area of the region between the curves $x = 4 - y^2$ and $x = -3y$.

   (a) $\frac{115}{6}$

   (b) $\frac{1}{4}$

   (c) $\frac{43}{2}$

   (d) $\frac{1}{9}$

   (e) $\frac{1}{12}$

   (f) 20

   (g) $\frac{125}{6}$

2. Compute

$$\int_0^{1/2} \frac{\arcsin(x)}{\sqrt{1-x^2}}\, dx\,.$$

   (a) $\frac{\pi^2}{36}$

   (b) $\frac{\pi}{36}$

   (c) $\frac{1}{36}$

   (d) $\frac{\pi}{6}$

   (e) $\frac{\pi^2}{72}$

   (f) $\frac{\pi}{72}$

   (g) $\frac{1}{72}$

3. Find the value of $\int_0^1 \ln(1+x^2)\,dx$.

   (a) $\ln 2$
   (b) $\dfrac{\pi}{8}$
   (c) $\pi - 2$
   (d) $\pi - \ln 2$
   (e) $\dfrac{\pi}{2} - 2 + \ln 2$
   (f) $2 - \ln 2$
   (g) $\dfrac{\pi}{4} + \ln 2$

4. An ant travels in the plane on a curve satisfying

$$\frac{dy}{dx} = \sqrt{4x^2 + 2x - \frac{3}{4}}$$

   when measured in inches. If the ant begins at the point $(1,1)$ and stops when the x-coordinate reaches 2, what total number of inches does the ant travel?

   (a) $\dfrac{3}{2}$
   (b) 2
   (c) $\dfrac{9}{4}$
   (d) 3
   (e) $\dfrac{7}{2}$
   (f) π
   (g) $\dfrac{\pi^2}{2}$

5. Which of the following functions solves the differential equation $y' = 4xy$?

(a) $y = e^{-4x}$

(b) $y = 4x$

(c) $y = e^{2x^2}$

(d) $y = e^{2x}$

(e) $y = 2x^2$

(f) $y = xe^{4x}$

6. Find the coefficient of x^5 in the Maclaurin series for the integral $\int_1^x \cos(t^2)\,dt$.

(a) $-\frac{1}{5}$

(b) $-\frac{1}{10}$

(c) $-\frac{1}{15}$

(d) $\frac{1}{15}$

(e) $\frac{1}{10}$

(f) $\frac{1}{5}$

7. Suppose that $y(x)$ is the solution to the differential equation $y'' = xy$ with initial conditions $y(0) = 1$ and $y'(0) = 0$. Which of the following is the closest to $y(1/2)$? [Hint: A series solution gives the required accuracy much more easily than does an Euler-type iteration.]

   (a) 0.98

   (b) 1.00

   (c) 1.02

   (d) 1.05

   (e) 1.20

   (f) 2.00

8. According to Taylor's remainder estimate, the maximum possible error in the use of $\sum_{n=0}^{4} \frac{x^n}{n!}$ to approximate e^x on the interval $[-1, 1]$ is which value?

   (a) $\dfrac{e}{240}$

   (b) $\dfrac{e}{120}$

   (c) $\dfrac{1}{48}$

   (d) $\dfrac{1}{24}$

   (e) $\dfrac{1}{24\,e}$

   (f) $\dfrac{1}{12\,e}$

9. Evaluate

$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{3^{2n}(2n+1)!}.$$

(a) $\dfrac{3\sqrt{3}}{2}$

(b) $\dfrac{\pi}{3-\pi}$

(c) $\sin(-\pi)$

(d) $\pi\, e^{-\pi/3}$

(e) $\pi\, e^{-\pi^2/9}$

(f) $\pi \sinh\left(\dfrac{\pi^2}{9}\right)$

(g) $3 \sinh\left(\dfrac{\pi^2}{9}\right)$

10. Let $f(x) = \sum_{n=0}^{\infty}(-1)^n n^2 x^n$. What is the third derivative, $f^{(3)}(x)$, evaluated at $x = 0$?

(a) -54

(b) -9

(c) $-3/2$

(d) $3/2$

(e) 9

(f) 54

11. What is the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{2 \cdot 5 \cdot 8 \cdots (3n-1)}{3 \cdot 7 \cdot 11 \cdots (4n-1)} (x+1)^n \,?$$

(a) 0

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

(e) $\frac{4}{3}$

(f) $\frac{3}{2}$

(g) 2

(h) ∞

12. Compute

$$\sum_{n=1}^{\infty} \frac{n}{3^{n-1}} \,.$$

(a) $\frac{9}{4}$

(b) $3e^{-3}$

(c) $\frac{4}{3}$

(d) $\sqrt{\frac{3}{2}}$

(e) 2

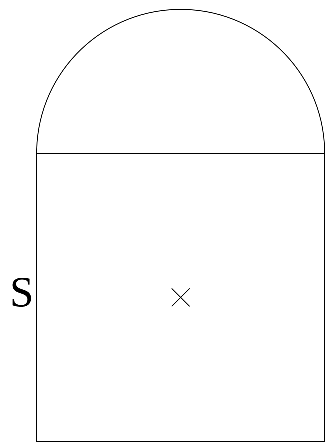
(f) $\frac{2}{3}$

(g) $\frac{3n}{2/3}$

Free answer questions

Each of these questions is worth 8 points. Please **show all work** that you wish to be considered for partial credit and **put a box** around the final answer to each (part of the) problem.

13. A standard US Mailbox has a cross-section shaped like a square of side S with a semicircle attached. Where is the center of mass of this cross-section? State your answer in coordinates with the origin at the center of the square.



14. Newton's Law of Cooling states that the rate at which a body changes temperature is proportional to the difference between its temperature and the temperature of the surrounding medium. Suppose that a body has an initial temperature of 252°F and that after one hour the temperature is 216°F. Assuming that the surrounding air is kept at a constant temperature of 72°F, determine the temperature of the body two hours after the starting time, in degrees Fahrenheit.

15. Solve
$$y' = \frac{\ln(x)}{x\,y}$$
with initial condition $y(1) = 2$.

16. Compute the first three terms of the Maclaurin series for $\sqrt{f}$ if the Maclaurin series for f begins $1 + ax + bx^2 + \cdots$.