Math 104 Final Exam, Fall 2003

1. (free response) Set $e^{2x} = P_n(x) + R_n(x)$, where P_n and R_n are the nth Taylor polynomial (at $a = 0$) and the remainder, respectively. Use Taylor's Theorem to prove that

$$\lim_{n\to\infty} R_n(x) = 0.$$

2. The coefficient of x^3 in the Maclaurin series for $xe^x \cos(x/2)$ is

ANSWER: 1/4

3. The interval of convergence of

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n 2^n}{\sqrt{n^2 + 3}}$$

is

ANSWER: $(-1/2, 1/2]$

4. Find

$$\int_0^{\infty} e^x \sin x \, dx$$

ANSWER: 1/2

5. The value of

$$\int_0^1 \frac{dx}{(4 - x^2)^{3/2}}$$

is

ANSWER:

$$\frac{1}{4\sqrt{3}}$$

6. The value of

$$\lim_{x\to 0^+} \frac{\sin x - x}{2x^3}$$

is

ANSWER: 1/12

7. In the partial freaction decomposition of

$$\frac{x + 4}{x^3 + 3x^2 - 10x}$$

the coefficient of $\frac{1}{x-2}$ is

ANSWER: 3/7

(In the next problem you are asked to determine the convergence or divergence of three separate sums. If you get two of the three correct you will receive 1/2 credit for that problem).

8. Let

$$A = \sum_{n=1}^{\infty} \frac{\ln^8 n}{n^2 - 1},$$

$$B = \sum_{n=1}^{\infty} \frac{2n}{n^2 + n + 1},$$

$$C = \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}.$$

Then

(a) A, B, C *converge* $\qquad$ (b) A, B *converge,* C *diverges*

(c) A, C *converge,* B *diverges* $\qquad$ (d) B, C *converge,* A *diverges*

(e) A *converges,* B, C *diverge* $\qquad$ (f) B *converges,* A, C *diverge*

(g) C *converges,* A, B *diverge* $\qquad$ (h) A, B, C *diverge*

ANSWER: (c): A,C converge and B diverges

9. The volume of the solid generated by revolving the region between $y = 0$, $x = 0$, $x = 2$ and $y = 1 + \frac{x^2}{4}$ about the line $x = 3$ is

ANSWER: 10π