**Math 104** **Fall 2002**
**Final Examination - Alt** **December 17, 2002**

Name ______________________________

Student ID number ______________________________

Professor ______________________ Section __________

T.A. ______________________ Recitation time __________

There are eighteen questions on this examination.
No calculators are allowed, but you may use a page with notes written on both sides.
Show your work in the space provided, and then write your answer in the box provided with each problem. You will not receive full credit for a problem unless your answer is written clearly in the box.

**Please do not write in the grid below:**

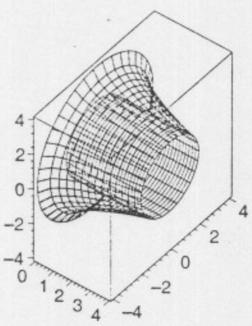
| | | | |
|---|---|---|---|
| 1. | | 11. | |
| 2. | | 12. | |
| 3. | | 13. | |
| 4. | | 14a. | |
| 5a. | | 14b. | |
| 5b. | | 15a. | |
| 6. | | 15b. | |
| 7. | | 16a. | |
| 8. | | 16b. | |
| 9. | | 17. | |
| 10. | | 18. | |
| | | | |

| Total |
|---|
| |

1. A scientist collects data that relate two variables, $x$ and $y$. Instead of plotting $y$ as a function of $x$, she plots $\log_3 y$ as a function of $x$, and gets a line whose slope is 2 and whose intercept on the vertical axis is $-1$. What equation describes $y$ as a function of $x$?

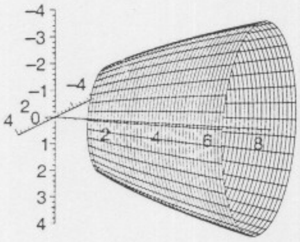
Answer:

2. Find the volume of the solid of revolution obtained by rotating the region bounded by the curve $y = 4 - \sqrt{x}$, the line $y = 2$ and the $y$-axis around the $x$-axis.




Answer:

3. The arc of the parabola $y = \sqrt{2x}$ from (2,2) to (8,4) is rotated around the $x$-axis. Find the surface area of the resulting surface.



Answer:

4. Solve the initial-value problem:

$$x\frac{dy}{dx} + 2y = x - 1, \qquad y(1) = 1$$

Answer:

5 (a). Because of air resistance, the velocity of a certain falling object decreases at a rate proportional to the square of the velocity. Write the differential equaiton that decribes the change in the velocity of the object, and then give the general solution of this differential equation.

Answer:

5 (b). Suppose the object in problem 5 (a) is initially falling at a rate of 16 meters per second, and after 5 seconds it has slowed to 10 meters per second. At what time will the object's velocity be 8 meters per second?

Answer:

6. Calculate $\displaystyle\int_1^\infty \frac{\ln x}{x^{3/2}}\,dx$, if it converges.

Answer:

7. Calculate $\displaystyle\int \frac{x}{x^2-5x-14}\,dx$.

Answer:

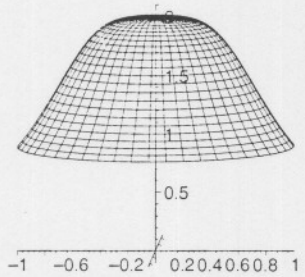
8. Evaluate $\displaystyle\int_0^{\pi/4} \sec^2\theta \tan^3\theta \, d\theta$.

Answer:

9. Sketch the graph of $y = (1+x)e^{-x}$ on the interval $[0, \infty)$. Be sure to indicate "interesting" points on the graph as well as any asymptotes it has. Draw your graph on the axes provided. In the answer box, list max/min points and inflection points.

Answer:

10. Calculate the volume of the solid obtained by rotating the part of the graph of $y = \dfrac{2}{\sqrt{4x^4+1}}$ for $x \in [0,1]$ around the $y$-axis.




Answer:

11. Investigate the convergence/divergence of the improper integral $\int_0^\infty \frac{dx}{\sqrt{x}(1+e^x)}$. (Do not attempt to evaluate the integral.)

Answer:

12. Find the limit of the sequence $\{\sqrt[n]{4^n + 3^n}\}$ or else explain why it does not converge.

Answer:

13. Investigate the convergence (absolute/conditional/divergence) of the series $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{1+n^2}$.

Answer:

14 (a). Use the first three non-zero terms of an appropriate series to give an approximation of

$$\int_0^{1/2} \cos(\sqrt{x})\, dx.$$

Answer:

14 (b). Give (with explanation) an estimate of the error (difference between your approximation and the actual value of the integral) in problem 14 (a).

Answer:

15 (a). Find the center and radius of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(-1)^n(1+n)}{2^n}(x-3)^n.$$

(In other words, find the largest *open* interval on which the series converges).

Answer:

15 (b). Investigate the convergence (absolute/conditional/divergence) of the series in 15 (a) at the endpoints of its interval of convergence.

Answer:

16 (a). Write the second-degree Taylor polynomial for $f(x) = \sqrt{x}$ centered at $a = 4$.

Answer:

16 (b). Use the polynomial from 16 (a) to estimate $\sqrt{4.5}$. Also, give an estimate of the error.

Answer:

Answer:

18. Does the series $\sum_{n=2}^{\infty} \frac{\ln(n!)}{n^2}$ converge or diverge? Explain your answer.

Answer: