UNIVERSITY *of* PENNSYLVANIA
Mathematics Department

MATH 104
FINAL EXAMINATION/FALL 2007

NAME: _____

YOUR PROFESSOR (CHECK ONE): ❑CROTTY ❑DETURCK ❑DRUMM ❑GOLUBEVA ❑KRASHEN ❑RIMMER ❑TEMKIN ❑VANERP

YOUR TA: _____

INSTRUCTIONS:
1. You have two hours for this examination.
2. You are permitted the use of a one page notes sheet (8.5x11, both sides).
3. Solve each problem in the space provided. Write the letter of your answer in the appropriate space on this page.
4. Show your work. A correct answer with no supporting work may receive little or no credit
5. Each problem is worth 10 points.
6. There are 20 problems; *you are to do all of them*.

Write the letters corresponding to your answers here:

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Score: _____

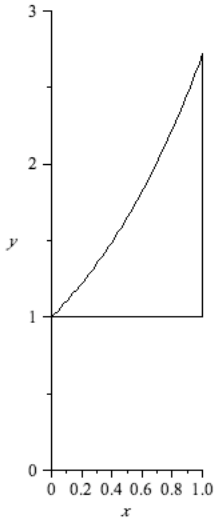
1. Find the average value of the function $f(x) = x\cos(2x)$ on the interval $0 \le x \le \frac{\pi}{4}$.

a) $\frac{1}{2\pi}$ b) $\frac{1}{2} - \frac{1}{\pi}$ c) $\frac{1}{2} - \frac{1}{2\pi}$ d) $1 - \frac{1}{\pi}$ e) $\frac{1}{4} - \frac{1}{2\pi}$ f) $\frac{1}{4} - \frac{1}{\pi}$

2. Find the volume of the solid obtained by rotating the region bounded by the curves $y = e^x$, $y = 1$, $x = 1$ about the x-axis.

a) $\frac{\pi}{2}\left(e^2 - 3\right)$ b) $\frac{\pi}{4}\left(e^2 - 1\right)$ c) $\pi\left(e^2 - 1\right)$

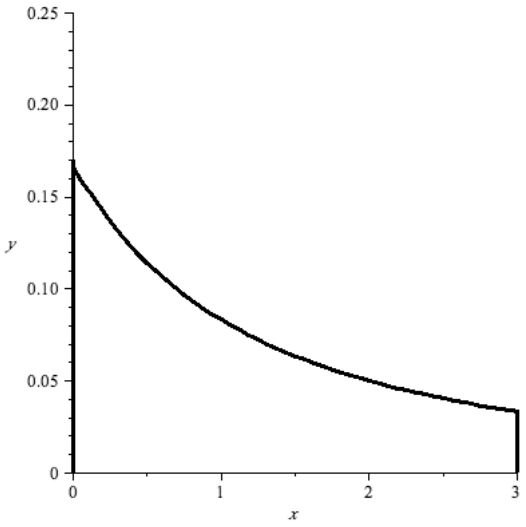
d) $\pi\left(e^2 - 3\right)$ e) $\frac{\pi}{2}\left(e^2 - 1\right)$ f) $\frac{\pi}{4}\left(e^2 - 3\right)$



3. What is the volume of the solid of revolution generated by rotating about the y-axis the region enclosed by the graph of $\dfrac{1}{x^2+5x+6}$, the x-axis and the lines $x = 0$ and $x = 3$.

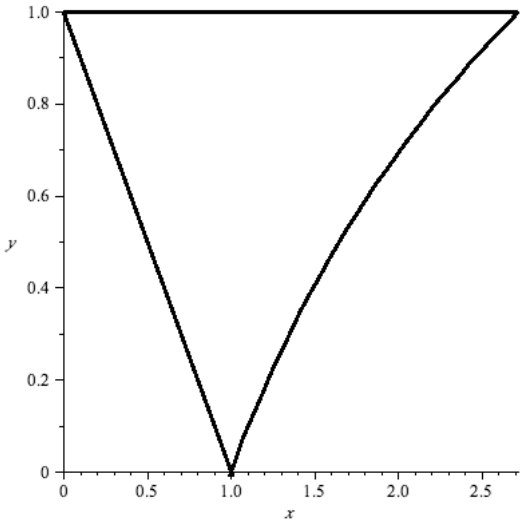
a) $2\pi(3\ln 6 - 2\ln 5)$ b) $2\pi(3\ln 6 - 2\ln 2)$ c) $2\pi(3\ln 3 - 2\ln 5)$

d) $2\pi(3\ln 5 - 2\ln 3)$ e) $2\pi(5\ln 2 - 2\ln 5)$ f) $2\pi(5\ln 2 - 3\ln 3)$



4. Calculate the area of the region bounded by the curves $y = \ln x$, $x + y = 1$ and $y = 1$.

a) $e - \frac{1}{4}$ b) $\frac{e}{2} + \frac{1}{2}$ c) $\frac{e}{2} - \frac{1}{2}$ d) $\frac{e}{2} - \frac{3}{2}$ e) $e - \frac{1}{2}$ f) $e - \frac{3}{2}$



5. Evaluate: $\int_0^e \frac{x^3}{3} \ln x \; dx$

a) $\frac{e^2}{2}$ b) $\frac{e^2}{2} - \frac{1}{2}$ c) $\frac{e^4}{36} + \frac{1}{32}$

d) $\frac{e^4}{36}$ e) $\frac{e^4}{16} - \frac{1}{4}$ e) $\frac{e^4}{16}$

6. Evaluate: $\int_4^\infty \frac{dx}{x^2 - 6x + 10}$

a) $\frac{\sqrt{3}}{2} + \frac{1}{2}$ b) $\frac{\sqrt{3}}{2} + \frac{1}{4}$ c) $\frac{\sqrt{3}}{4} + \frac{1}{4}$ d) $\frac{\pi}{2}$ e) $\frac{\pi}{4}$ f) diverges

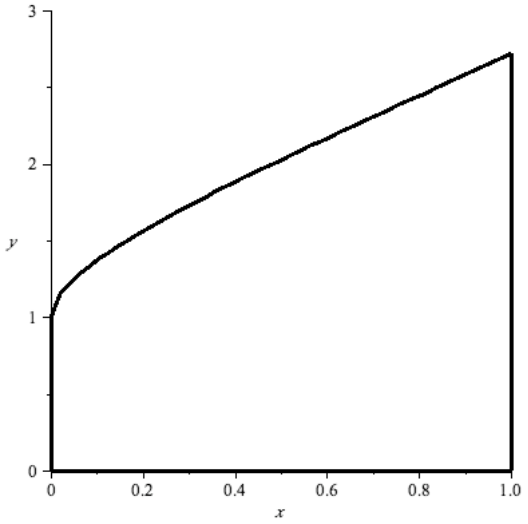
7. Evaluate: $\int_0^2 \sqrt{16 - x^2}\ dx$

a) $\frac{\sqrt{3}}{2} + \frac{8\pi}{2}$ b) $\sqrt{3} + \frac{8\pi}{3}$ c) $2\sqrt{3} + \frac{8\pi}{3}$

d) $\frac{\sqrt{3}}{2} + \frac{4\pi}{3}$ e) $\sqrt{3} + \frac{4\pi}{3}$ f) $2\sqrt{3} + \frac{4\pi}{3}$

8. Find the volume of the solid of revolution obtained by revolving the region between the graph of $y = e^{\sqrt{x}}$ and the x-axis for $0 \le x \le 1$ around the x-axis.

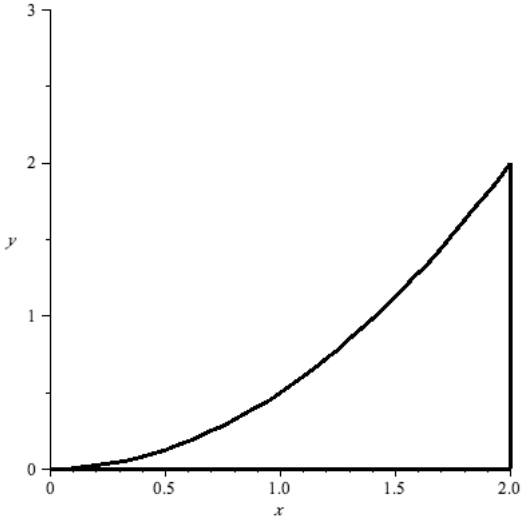
a) $\frac{\pi}{2}(e-1)$ b) $\frac{\pi}{2}(e^2-1)$ c) $\frac{\pi}{2}(e+1)$ d) $\frac{\pi}{2}(e^2+1)$ e) $\frac{\pi}{2}(e-3)$ f) $\frac{\pi}{2}(e^2-3)$



9. Find the surface area of the surface obtained by revolving the part of the curve $y = \frac{1}{2}x^2$ for $0 \le x \le 2$ around the *y*-axis.

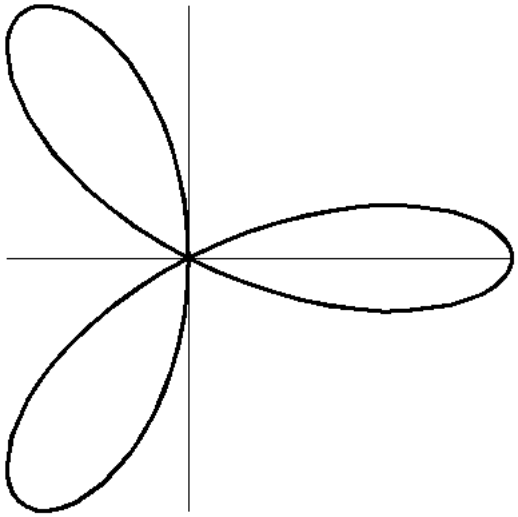
a) $\frac{\pi}{3}(5\sqrt{5}-1)$ b) $\frac{\pi}{3}(\sqrt{5}-1)$ c) $\frac{2\pi}{3}(5\sqrt{5}-1)$

d) $\frac{2\pi}{3}(\sqrt{5}-1)$ e) $\frac{\pi}{2}(5\sqrt{5}-1)$ f) $\frac{\pi}{2}(\sqrt{5}-1)$



10. Find the area inside *one* leaf (i.e., one loop) of the graph of $r = 4\cos 3\theta$.

a) $\frac{2\pi}{3}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{2}$ d) $\frac{4\pi}{3}$ e) $\frac{7\pi}{4}$ f) 2π



11. Find the limit: $\lim_{x\to 0} \dfrac{\sin^3 x - x^3}{(\cos x - 1)^2}$

a) $\dfrac{1}{e}$ b) π c) -1 d) 0 e) 1 f) no finite limit

12. Find the limit of the sequence $\left\{\left(\frac{1+n}{2+n}\right)^n\right\}$.

a) $\frac{1}{2}$ b) $\frac{1}{e}$ c) 0 d) 1 e) e f) sequence diverges

13. Determine if the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{3^n + 4^n}{7^n}$$

a) 1 b) $\frac{1}{7}$ c) $\frac{5}{12}$ d) $\frac{12}{7}$ e) $\frac{25}{12}$ f) divergent

14. How many of the following series converge?

$$\sum_{n=1}^{\infty}\frac{\sqrt{n}+1}{n+2} \qquad \sum_{n=1}^{\infty}\frac{2^n+n^2}{3^n+n^3} \qquad \sum_{n=1}^{\infty}\frac{n^2+3}{(n+4)^2} \qquad \sum_{n=2}^{\infty}\frac{\ln n}{n^2} \qquad \sum_{n=2}^{\infty}\frac{n^2}{\ln n}$$

a) none b) one c) two d) three e) four f) five

15 Which statement is true of the following series?

- (I) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1} = \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \ldots$
- (II) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n!)^2}{(2n)!} = \frac{1}{2} - \frac{1}{6} + \frac{1}{20} - \ldots$
- (III) $\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!} = 1 - 2 + \frac{9}{2} \ldots$

A) (I) converges conditionally, (II) converges absolutely, (III) diverges
B) (I) converges absolutely, (II) converges conditionally, (III) diverges
C) (I) and (II) converge absolutely, (III) converges conditionally
D) (I) and (II) converge absolutely, (III) diverges
E) (I) and (III) diverge, (II) converges conditionally
F) (I) and (III) converge conditionally, (II) converges absolutely

16. What is the largest open interval on which the series $\sum_{n=1}^{\infty} \frac{n(x-5)^n}{n^3+4^n}$ converges?

a) (–4, 4) b) (–4, 5) c) (4, 6) d) (1, 9) e) (4, 5) f) (–4, 14)

17 Which of the following is the beginning of the Maclaurin series for $\arctan(x^2)$?

a) $x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \ldots$

b) $\frac{x^2}{3} - \frac{x^6}{6} + \frac{x^{10}}{9} - \frac{x^{14}}{12} + \ldots$

c) $x^2 - 2x^4 + 3x^6 - 4x^8 + \ldots$

d) $1 + 2x^2 + 3x^4 + 4x^6 + \ldots$

e) $x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - \frac{x^{14}}{7} + \ldots$

f) $x^2 + \frac{x^6}{2} + \frac{x^{10}}{3} + \frac{x^{14}}{4} + \ldots$

18. Compute the arclength of the graph of the function $y = \frac{2}{3}x^{\frac{3}{2}}$ for $0 \le x \le 4$.

a) $\frac{2}{3}\left(5^{\frac{3}{2}} - 1\right)$ b) $\frac{3}{2}\left(5^{\frac{1}{2}} + 1\right)$ c) $\frac{2}{3}\left(5^{\frac{3}{2}} + 1\right)$

d) $\frac{3}{2}\left(5^{\frac{1}{2}} - 1\right)$ e) 4 f) $\frac{8}{15}\sqrt{265}$

19. Evaluate $\int_2^3 \frac{x^3}{x^2+1}dx$.

a) $\frac{1-\sqrt{2}}{2}$ b) $\frac{5-\ln 2}{2}$ c) $\frac{\sqrt{2}}{2}$ d) $\frac{\pi}{2}$ e) $\frac{1+\sqrt{2}}{2}$ f) $\frac{1}{2}+\frac{\pi}{2}$

20. One of the two following integrals converges.

$$\int_0^{\infty} e^{-x} \sin x \; dx; \quad \int_{-\infty}^{0} e^{-x} \sin x \; dx$$

The value of the convergent integral is:

a) $\frac{1}{2}$ b) $\frac{1}{2} - e$ c) $-e$ d) e e) $e - \frac{1}{2}$ f) $-\frac{1}{2}$