Math 104
Makeup Final Exam
May 2, 2007

Name ____

Penn ID ____

Circle exactly one professor/TA combination:

Professor:	Dr. Ackerman	Dr. Popa	Dr. van Erp	Dr. Ward	Dr. Ward
TA:	Andrew Rupinski	Jen Hom	Andrew Bressler	Alexa Mater	Tim DeVries
time:	1 PM, MWF	10:30 AM, TR	3 PM, TR	10 AM, MWF	10 AM, MWF

<u>Write all answers (A, B, C, D, E, F) in the spaces provided below!</u>

1. ________	6. ________	11. ________	16. ________
2. ________	7. ________	12. ________	17. ________
3. ________	8. ________	13. ________	18. ________
4. ________	9. ________	14. ________	19. ________
5. ________	10. ________	15. ________	20. ________

Score: ________ (100 points possible)

1. The testing booklet contains 20 questions.
2. No calculators are permitted.
3. One piece of paper (8.5 in. by 11 in.) is permitted, with writing on both sides allowed.
4. There is no penalty for guessing.
5. No partial credit will be given.
6. Write all calculations on the pages provided. Extra pages are available if needed.

1. Find the limit.

$$\lim_{n\to\infty} \frac{2n+\cos(3n)}{3n+\sin(2n)}$$

A.) 0 B.) 1 C.) $\cos(3)/\sin(2)$ D.) $2/3$ E.) $3/2$ F.) no limit exists

2. Find the volume of the solid obtained by rotating the region bounded by the curves

$y = e^{x^2}$ and $y = 0$ and $x = 0$ and $x = 2$

about the y-axis.

A.) $4\pi e^4$ B.) $2\pi e^4$ C.) $2\pi(e^4 - 1)$ D.) $\pi(e^4 - 1)$ E.) $\pi\sqrt{e}$ F.) πe

3. Find an equation of the tangent to the curve

$$x = 2(t - \sin t)\,, \qquad\qquad y = 2(1 - \cos t)$$

at the point

$$(x, y) = \left(\frac{2\pi}{3} - \sqrt{3},\ 1\right).$$

A.) $y = \frac{\sqrt{3}}{4}x + \frac{7}{4} - \frac{\pi}{6}\sqrt{3}$

B.) $y = \frac{\sqrt{3}}{3}x + 2 - \frac{2\pi}{9}\sqrt{3}$

C.) $y = \frac{\sqrt{3}}{2}x + \frac{5}{2} - \frac{\pi}{3}\sqrt{3}$

D.) $y = \sqrt{3}\,x + 4 - \frac{2\pi}{3}\sqrt{3}$

E.) $y = 2\sqrt{3}\,x + 7 - \frac{4\pi}{3}\sqrt{3}$

F.) $y = 3\sqrt{3}\,x + 10 - 2\pi\sqrt{3}$

4. Determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}.$$

A.) $[-1, 3)$

B.) $(-1, 3]$

C.) $[-1, 5)$

D.) $(-1, 5]$

E.) $[3, 5)$

F.) $(3, 5]$

5. Find the area of the region enclosed by the curves

$$y = e^x \qquad \text{and} \qquad y = e^{-x} \qquad \text{and} \qquad x = 2\,.$$

A.) $3 - e$ B.) $-1 + 2\ln 2$ C.) $e^2 - e^{-2}$ D.) $e^2 - 1$ E.) $5 - e^{-2}$ F.) $e^2 + e^{-2} - 2$

6. Find the first five terms of the Maclaurin series for

$$f(x) = \frac{x}{1-x^3}\,.$$

A.) $1 + x + x^2 + x^3 + x^4 + \cdots$

B.) $1 + x^3 + x^6 + x^9 + x^{12} + \cdots$

C.) $x + x^4 + x^7 + x^{10} + x^{13} + \cdots$

D.) $x^3 + x^4 + x^5 + x^6 + x^7 + \cdots$

E.) $x^3 + x^6 + x^9 + x^{12} + x^{15} + \cdots$

F.) $x + x^2 + x^3 + x^4 + x^5 + \cdots$

7. Evaluate the integral.

$$\int_0^1 \frac{dx}{x^{1/3}(x^{2/3}+1)}$$

A.) 0 B.) 2/3 C.) 1 D.) 3/2 E.) $\ln(2)$ F.) $\frac{3}{2}\ln(2)$

8. Determine whether the series is convergent or divergent. If the series is convergent, find its sum.

$$\sum_{n=0}^{\infty} e^{-3n}$$

A.) $\frac{1}{e-1}$ B.) $\frac{e}{e-1}$ C.) $\frac{1}{1-e^3}$ D.) $\frac{1}{e^3-1}$ E.) $\frac{e^3}{e^3-1}$ F.) divergent

9. Consider the following two series:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n} \qquad \text{and} \qquad \sum_{n=2}^{\infty} \frac{(-1)^n}{ne^n}$$

Which of the following is true?

A.) both series are absolutely convergent

B.) both series are conditionally convergent

C.) both series are divergent

D.) one series is absolutely convergent, and one series is conditionally convergent

E.) one series is absolutely convergent, and one series is divergent

F.) one series is conditionally convergent, and one series is divergent

10. Find the volume of the solid obtained by rotating the region bounded by the curves

$y = e^x$ and $y = 0$ and $x = 0$ and $x = 1$

about the y-axis.

A.) $2\pi e$ B.) 2π C.) $2\pi(e-1)$ D.) πe E.) π F.) $\pi(e-1)$

11. Evaluate the integral.

$$\int_3^4 \frac{4}{x^2-4}\,dx$$

A.) $\ln(12/5)$ B.) $\ln(5/3)$ C.) $\ln(1/3)$ D.) $4\ln(7)$ E.) $\ln(4/3)$ F.) $\ln(2/15)$

12. Find the area of the surface obtained by rotating the curve

$$y = 2x^2 + 1\,, \qquad 3 \le y \le 9$$

about the y-axis.

A.) $\frac{\pi}{48}(65\sqrt{65} - 1)$

B.) $\frac{2\pi}{3}(5\sqrt{5} - 1)$

C.) $\frac{\pi}{12}(17\sqrt{17} - 5\sqrt{5})$

D.) $\frac{\pi}{24}(65\sqrt{65} - 17\sqrt{17})$

E.) $\pi\left(\frac{99}{10} - \frac{5\sqrt{5}}{6}\right)$

F.) $\frac{2\pi}{3}(5\sqrt{5} - 2\sqrt{2})$

13. Consider the following three series:

$$\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}} \qquad \text{and} \qquad \sum_{n=1}^{\infty} \frac{1}{n^2+\sqrt{n}} \qquad \text{and} \qquad \sum_{n=1}^{\infty} \frac{1}{n^2+n}$$

Which of the following is true?

A.) only the first series converges

B.) only the second series converges

C.) only the third series converges

D.) both the first and second series converge

E.) both the first and third series converge

F.) both the second and third series converge

14. Find the first few terms of the Maclaurin series for

$$f(x) = \int \frac{\sin x}{x}\, dx\,.$$

A.) $C + x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

B.) $C + 1 - \frac{x^2}{(3)(3!)} + \frac{x^4}{(5)(5!)} - \frac{x^6}{(7)(7!)} + \cdots$

C.) $C + 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots$

D.) $C + 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

E.) $C + x - \frac{x^3}{(3)(3!)} + \frac{x^5}{(5)(5!)} - \frac{x^7}{(7)(7!)} + \cdots$

F.) $C + x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

15. Find the volume of the solid obtained by rotating the region bounded by the curves

$$y = x^{3/2} \qquad \text{and} \qquad y = 0 \qquad \text{and} \qquad x = 1$$

about the y-axis.

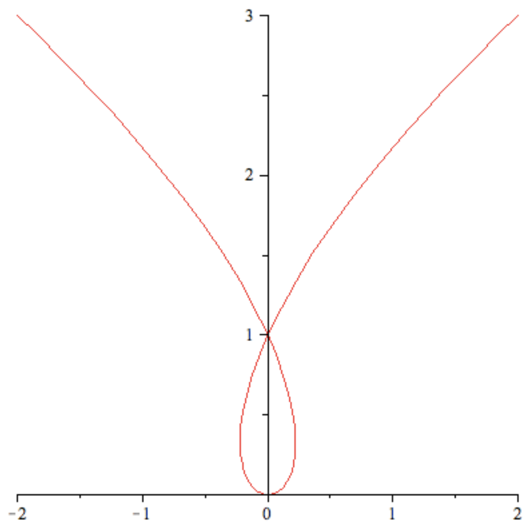
A.) $2\pi/5$ B.) $\pi/2$ C.) $5\pi/4$ D.) $4\pi/3$ E.) $5\pi/2$ F.) $4\pi/7$

16. A curve C is defined by the parametric equations

$$x = t - 3t^3\,, \qquad y = 3t^2\,.$$

The curve is given below. Notice that part of the curve contains a loop. Find the length of the loop.

A.) 0 B.) 8/9 C.) $\frac{4\sqrt{3}}{3}$ D.) $\frac{16\sqrt{3}}{5}$ E.) $\frac{8\sqrt{3}}{15}$ F.) 16/45



17. The curve

$$r = 2\sin(3\theta)$$

is a calculus flower with three leaves. Find the area of the region contained in the flower.

A.) $\pi/6$ B.) $\pi/4$ C.) $\pi/3$ D.) $\pi/2$ E.) π F.) 2π

18. Evaluate the integral.

$$\int_0^{1/2} \sqrt{1-4x^2}\, dx$$

A.) $\pi/8$ B.) $\pi/6$ C.) $\pi/4$ D.) $\pi/3$ E.) $\pi/2$ F.) π

19. Evaluate the integral.

$$\int_0^1 x^2 e^{2x}\,dx$$

A.) $\frac{1}{4}(e^2 - 1)$ B.) $\frac{1}{4}(5e^2 - 1)$ C.) $\frac{1}{4}(3e^2 + 1)$ D.) $\frac{1}{2}e^2 - 1$ E.) $-1/2$ F.) $\frac{5}{2}e^2 - 4$

20. Find the limit.

$$\lim_{x\to 0} \frac{\sin x + \cos x - 1 - x}{x^2}$$

A.) $-1/2$ B.) 0 C.) 1 D.) 2 E.) π F.) the limit does not exist