MATH 103 – Sample Final Exam 2

1. At what value or values of x is the function $f(x) = \begin{cases} |x+1| - 1 & \text{if } x < 0 \\ x^2 + x & \text{if } 0 \le x < 1 \\ 3 - x & \text{if} x \ge 1 \end{cases}$ not differentiable?

   (a) -1 (b) 1 (c) $0,\ 1$ (d) $-1,\ 0$
   (e) $-1,\ 0,\ 1$ (f) $-1,\ 1$ (g) 0 (h) differentiable everywhere

2. Which of the following limits is *not* equal to zero?

   (a) $\lim_{x\to\infty} x^5 e^{-x}$ (b) $\lim_{x\to 0} \sqrt{x}\ln x$ (c) $\lim_{x\to 0} \frac{\sin^2(x^2)}{x^3}$ (d) $\lim_{x\to 2} \int_2^x \cos\sqrt{t^5+1}\,dt$
   (e) $\lim_{x\to 0} \frac{e^x - 1}{x}$ (f) $\lim_{x\to 0} \frac{\cos 2x - 1}{x}$ (g) $\lim_{x\to\infty} \frac{x^3 + 2x - 3}{x^4 - 5x + 1}$ (h) $\lim_{x\to 0} x \sin\frac{1}{x}$

3. What are the domain and range of the function $f(x) = \log_3(9 - x^2)$?

   (a) Domain: $(-\infty,\infty)$, Range: $(-\infty, 0)$ (b) Domain: $(-\infty, 3)$, Range: $(-\infty, 2]$
   (c) Domain: $(-3,3)$, Range: $(-\infty,\infty)$ (d) Domain: $(-3,3)$, Range: $(-\infty, 2]$
   (e) Domain: $(-3,\infty)$, Range: $[2,\infty)$ (f) Domain: $(-3,\infty)$, Range: $(-\infty, 2]$
   (g) Domain: $[-3,3]$, Range: $(-\infty, 2]$ (h) Domain: $[-3,3]$, Range: $(-\infty, 2)$

4. Find the equation of the tangent line to the curve $y = \frac{x}{\sqrt{2 - x^2}}$ at the point $(1,1)$.

   (a) $y = 2x + 1$ (b) No tangent line exists (c) $y = 2x - 1$ (d) $y = 1$
   (e) $y = 3x - 2$ (f) $y = -x + 2$ (g) $y = -2x + 3$ (h) $y = -2x + 1$

5. What is the output of the following Maple statement?

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> subs(x=1,diff(ln(2*x)+x^(ln(2)),x));
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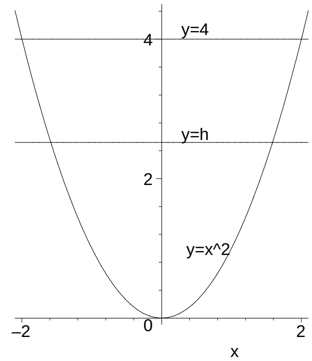
   (a) 1 (b) $2\ln 2$ (c) $-1 + \ln 2$ (d) $1 + \ln 2$
   (e) $\frac{1}{\ln 2} + 1$ (f) $\frac{1}{\ln 2} + \ln 2$ (g) $1 + \frac{1}{2}\ln 2$ (h) 0

6. Suppose f is an *even* function that satisfies $\int_0^2 f(x)\,dx = -6$ and $\int_1^2 f(x)\,dx = -5$. Find $\int_{-1}^1 f(x)\,dx$.

   (a) 1 (b) -1 (c) 2 (d) -2
   (e) 12 (f) -12 (g) 22 (h) -22

7. Consider the region bounded below by the graph of $y = x^2$ and above by the line $y = 4$ The horizontal line $y = h$ cuts this region into two pieces of equal area. What is h?



(a) $h = 2$ (b) $h = 2\sqrt[3]{2}$ (c) $h = \sqrt[3]{10}$ (d) $h = 3$
(e) $h = 2\sqrt{2}$ (f) $h = \sqrt{2} + \sqrt{3}$ (g) $h = 3\sqrt{2} - \sqrt{3}$ (h) $h = \sqrt{5}$

8. A rocket that is ascending vertically at a rate of 3 miles per second is being tracked by a camera that is on the ground, 1 mile away from the rocket's launch point. Let θ be the angle the camera makes with the horizontal. At what rate is θ changing when the rocket's altitude is 1 mile?

(a) $\frac{\pi}{4}$ rad/sec (b) 1 rad/sec (c) 1.5 rad/sec (d) 2 rad/sec
(e) 2.5 rad/sec (f) 3 rad/sec (g) 3.5 rad/sec (h) $\frac{\pi}{8}$ rad/sec

9. Evalutate the definite integral $\displaystyle\int_0^{\pi^2/4} \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$.

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) 2
(e) $2\sqrt{2}$ (f) $\frac{\sqrt{2}}{2}$ (g) $\frac{\pi\sqrt{2}}{2}$ (h) $\frac{1}{2}$

10. Let $f(x)$ be a continuous, one-to-one function on $0 \le x \le 1$ with $f(0) = 0$ and $f(1) = 1$. If $\int_0^1 f(x)\,dx = 2$, then what is the value of $\int_0^1 f^{-1}(y)\,dy$? (*Hint*: Draw a picture!)

(a) 0.8 (b) 0.6 (c) 0.4 (d) 0.2 (e) 1.25
(f) 1.667 (g) 2.5 (h) Cannot be determined without more information.

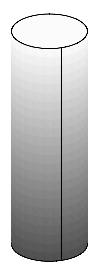
11. Let R be the region between the x-axis and the curve $y = \sin^{3/2} x$ for $0 \leq x \leq \pi$. If R is rotated around the x-axis, then the volume of the resulting solid is:

(a) $\dfrac{4\pi}{3}$ (b) 2π (c) $2\pi/3$ (d) $\pi^{3/2}$
(e) $\dfrac{7\pi}{3}$ (f) 3π (g) $\dfrac{2\pi^{3/2}}{3}$ (h) 4π

12. When a child was born, her grandparents placed \$1000 in a savings account at 10% interest compounded continuously, to be withdrawn at age 20 to help pay for college. How much money will be in the account at the time of withdrawal?

(a) $1000e$ dollars (b) $500e$ dollars (c) $500e^2$ dollars (d) $2000e^2$ dollars
(e) $4000e$ dollars (f) $2000e$ dollars (g) $1000e^2$ dollars (h) $4000e^2$ dollars

13. A closed tank is to be constructed in the shape of a right circular cylinder. It is made from three sheets of metal (top, bottom and side) with welded seams (the dark lines). Because the tank may eventually leak at the seams, they should be as short as possible. Assuming that the tank is to hold 1000 liters, find the radius and height in meters that minimizes the total length of the seams. (Note that 1000 liters = 1 cubic meter).



14. A population of aphids (little bugs that can damage or destroy a plant) on a rose plant increases at a rate proportional to the number present (i.e., exponentially). In three days, the population of aphids went from 800 to 1400.

(a) Write an equation for the population of aphids at anyu time t in days, where $t = 0$ is the day there were 800 aphids.

(b) How long does it take for the population to get 10 times as large?

(c) What was the population on the day before there were 800 aphids?

(d) Find the derivative of your population function on the fifth day ($t = 5$). Give the correct units for this quantity, and an interpretation of this number in practical terms.

15. In each case below, decide whether a function with the given properties can exist. If yes, then sketch a possible graph of such a function; if no, explain briefly why not.

A. $f(x) < 0$ and $f'(x) > 0$ for all x
B. $f(x) < 0$ and $f'(x) < 0$ for all x
C. $f''(x) > 0$ and $f(x) < 0$ for all x
D. $f(x) > 0$, $f'(x) > 0$ and $f''(x) < 0$ for all $x > 0$

16. Let $f(x) = 1 - kx^2$, where k is a real constant. It is easy to see (in the graph below) that for $k \leq 0$ and even for small positive values of k, the point on the graph of $y = f(x)$ closest to the origin is $(0, 1)$. It is equally easy to see (in the graph below) that for large positive values of k, $(0, 1)$ is *not* the closest point on the graph to the origin. What is the largest value of k for which the point on the graph of $y = f(x)$ closest to the origin is $(0, 1)$?