MATH 103 – Sample Final Exam 1

1. The domain of the function $f(x) = \sqrt{4 - \sqrt{x}}$ is

   (a) $x \leq \sqrt{8}$ (b) $0 \leq x \leq 16$ (c) $x \geq 0$ (d) $-4 \leq x \leq 4$ (e) all real x

2. What is the output of the following Maple statement?

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> limit((x^3-1)/(x^2-1),x=1);
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   (a) 3 (b) 5/2 (c) 3/2 (d) 0 (e) undefined

3. The function $f(x) = x^3 + 3x - 1$ has a root:

   (a) between -1 and 0 (b) between 0 and 1 (c) between 1 and 2
   (d) between 2 and 3 (e) between 3 and 4

4. $\lim_{x \to 0} x \cot(2x) =$

   (a) does not exist (b) 0 (c) 1/2 (d) 1 (e) 2

5. What is the output of the following Maple statement?

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> subs(x=9,diff(sqrt(1+sqrt(x)),x));
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   (a) 0 (b) 1/3 (c) -1/3 (d) 1/24 (e) 1/48

6. $\lim_{h \to 0} \dfrac{\sqrt[3]{8+h} - 2}{h}$ is

   (a) ∞ (b) $\dfrac{1}{24}$ (c) $\dfrac{1}{12}$ (d) $\dfrac{2}{3}$ (e) 0

7. The two tangents that can be drawn from the point (3,5) to the parabola $y = x^2$ have slopes

   (a) 2 and $-\dfrac{1}{2}$ (b) 2 and 4 (c) 1 and 5 (d) 2 and 10 (e) 0 and 4

8. An asteroid hits the Atlantic Ocean and creates an expanding circular wave. If the area enclosed by this wave increases at the rate of 200 km^2/min, how fast is the *diameter* of the wave expanding when its *radius* is 20 km?

   (a) $\pi/10$ km/min (b) $\pi/5$ km/min (c) 5π km/min
   (d) $5/\pi$ km/min (e) $10/\pi$ km/min

9. Evalutate the definite integral $\int_0^2 (2x - \sqrt{2x})\, dx$.

(a) 0 (b) 8 (c) 20/3 (d) 4/3 (e) 12

10. $\int_0^{\sqrt{\pi/2}} x\cos(x^2)\, dx =$

(a) 0 (b) 1/2 (c) 1 (d) $\pi/2$ (e) $\sqrt{\pi}$

11. The volume of the solid obtained by rotating the region in the plane bounded by the curves $y = x - x^2$ and $y = 0$ around the line $x = 2$ is

(a) $\pi/2$ (b) π (c) $2\pi/3$ (d) $3\pi/4$ (e) $4\pi/3$

12. We have

$$\arctan(3) = \int_0^3 \frac{1}{1+x^2}\, dx.$$

Using the trapezoid method with $n = 3$ intervals, give an approximation for arctan(3).

(a) $e - 1$ (b) 4/3 (c) 5/4 (d) 2 (e) 35/24

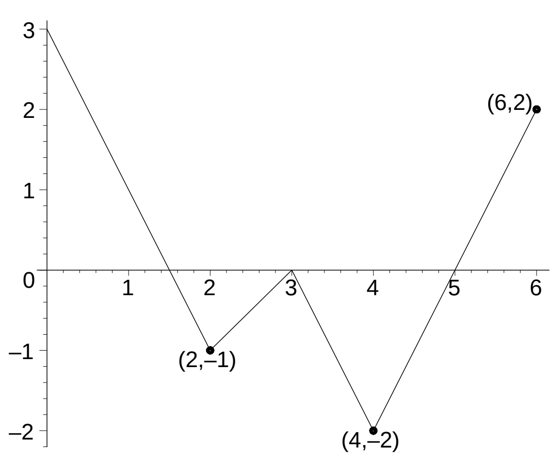
13. A bacterial culture grows exponentially from 100 to 400 grams in 10 hours.

   (a) How much was present after 3 hours?

   (b) What was the instantaneous growth rate of the mass of the culture at time = 5 hours? Express your answer in grams/hour.

   (c) What was the average mass of the culture over the 10 hours?

14. The graph below is the graph of $g'(x)$ (the derivative of the function $g(x)$). Suppose we also know that $g(0) = 10$.



(a) Fill in the following table:

x	0	1	2	3	4	5	6
$g(x)$	10						

(b) What are the maximum and minimum values of $g(x)$ for $x \in [0, 6]$, and where do they occur?

(c) Where is the graph of $g(x)$ concave up?

(d) For what values of x does the graph of $g(x)$ have an inflection point?

(e) Sketch the graph of $g(x)$.

15. A landscape architect plans to enclose a 3000 square foot rectangular region in a botanical garden. She will use shrubs costing \$25 per linear foot along three sides of the region, and fencing costing \$20 per linear foot along the fourth side. Find the dimensions of the region that minimize the total cost for the shrubs and fence.

16. Let $g(x) = \dfrac{\ln x}{x}$ for $x > 0$.

(a) For what values of x does $g(x)$ have a (local) maximum, minimum, or inflection point?

(b) Compute the range of $g(x)$.

(c) For what values of c does the equation $\ln x = cx$ have at least one solution?

(d) For what values of c does the equation $\ln x = cx$ have *more than one* solution?

(e) For that values of $a > 0$ does the equation $a^x = x$ have a solution?