FINAL EXAM

Math 103
5/5/2014

Name: _____

ID: _____

"My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this"

Signature: _____

Read all of the following information before starting the exam:

- Check your exam to make sure all 11 pages are present.
- You may use writing implements and a single handwritten sheet of 8.5"x11" paper.
- NO CALCULATORS.
- Show all work, clearly and in order, if you want to get full credit. You must show work to receive full credit.
- Good luck!

1	6		9	6	
2	6		10	6	
3	6		11	6	
4	6		12	6	
5	6		13	9	
6	6		14	10	
7	6		15	9	
8	6				
Total	100				

Name: _____

_____ **1.** Find $\int_1^4 \frac{1}{2x} dx$.

a. 0
b. $\ln 2$
c. $2 \ln 2$
d. $\ln 2 - 1$
e. $\ln 4 - 1$
f. $\frac{1}{2} \ln 2$
g. $\frac{1}{4} \ln 2$
h. DNE

_____ **2.** If $g(x) = \sqrt{1 - x^2} + x \sin^{-1} x$, find $g'(\sqrt{2}/2)$.

a. 0
b. $\pi/6$
c. $\pi/4$
d. $\pi/3$
e. 1
f. $\pi/2$
g. π
h. -1

_____ **3.** At which values of x does the function

$$f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x < 1 \\ \frac{1}{2-x} & \text{if } 1 \le x \end{cases}$$

fail to be differentiable?

a. $\{0, 1, 2\}$
b. $\{1, 2\}$
c. $\{0, 1\}$
d. $\{0, 2\}$
e. $\{0\}$
f. $\{1\}$
g. $\{2\}$
h. The function is differentiable everywhere

_____ **4.** If $f(x) = \frac{(x^2+1)(x+1)\cos x}{e^x(x+3)}$, find $f'(0)$.

a. -3
b. -1
c. $-1/3$
d. $-1/9$
e. 3
f. 1
g. $1/3$
h. $1/9$

_____ **5.** Find $\lim_{x\to 0}(2-e^x)^{1/x}$.

a. $-e$
b. -1
c. $-1/e$
d. DNE, goes to $-\infty$
e. e
f. 1
g. $1/e$
h. DNE, goes to $+\infty$

_____ **6.** Find $\lim_{x\to 0}\frac{2(x+1)}{e^x}$.

a. -2
b. -1
c. $-1/e$
d. DNE, goes to $-\infty$
e. 2
f. 1
g. $1/e$
h. DNE, goes to $+\infty$

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_____ **7.** 3 grams of radioactive polonium are stored in a container. Every year, 12.5% (that is, 1/8th) of the polnium decays away. What is the half life of this polonium (that is, how many years until there are 1.5 grams of the polonium left)?

a. 3
b. 4
c. $\ln 2 - \ln(7/8)$
d. $-8 \ln .5$
e. $2 \ln(7/8)$
f. $\frac{\ln 2}{\ln(8/7)}$
g. $\frac{8 \ln 2}{7}$
h. $\ln(7/8)$

_____ **8.** Find $\int \frac{1+x}{1+x^2} dx$.

a. $\tan^{-1} x + C$
b. $\frac{1}{2} \ln(1 + x^2) + C$
c. $\ln(1 + x^2) + C$
d. $\tan^{-1}(1 + x^2) + C$
e. $2 \tan^{-1}(1 + x^2) + C$
f. $2 \tan^{-1} x^2 + C$
g. $\tan^{-1} \ln(1 + x^2) + C$
h. $\tan^{-1} x + \frac{1}{2} \ln(1 + x^2) + C$

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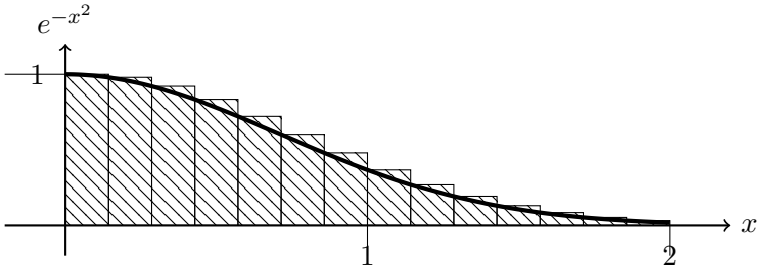
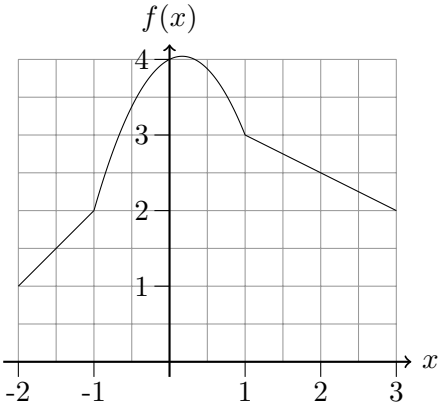
_____ **9.** This picture shows an estimate of $\int_0^2 e^{-x^2}\,dx$ using 14 rectangles. When k is an integer between 1 and 14, what is the area of the k-th rectangle?



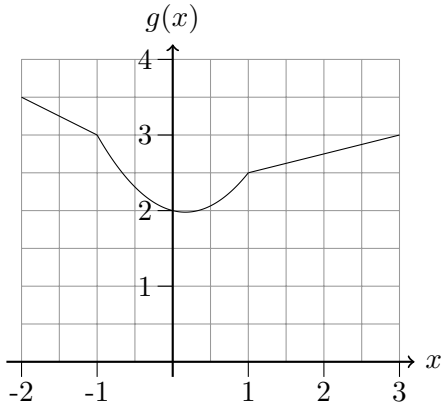
a. $\frac{1}{14}e^{-k^2}$
b. $\frac{1}{14}e^{-(k-1)^2}$
c. $\frac{1}{7}e^{-k^2}$
d. $\frac{1}{7}e^{-(k-1)^2}$
e. $\frac{1}{14}e^{-(k/14)^2}$
f. $\frac{1}{14}e^{-((k-1)/14)^2}$
g. $\frac{1}{7}e^{-(k/7)^2}$
h. $\frac{1}{7}e^{-((k-1)/7)^2}$

_____ **10.**

This is the graph of a function $f(x)$:



Here is the graph of a related function $g(x)$:

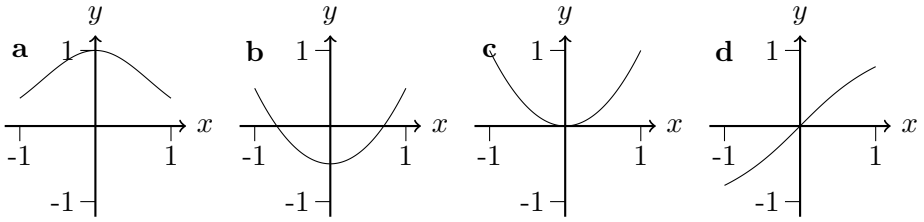


What is a formula for $g(x)$?

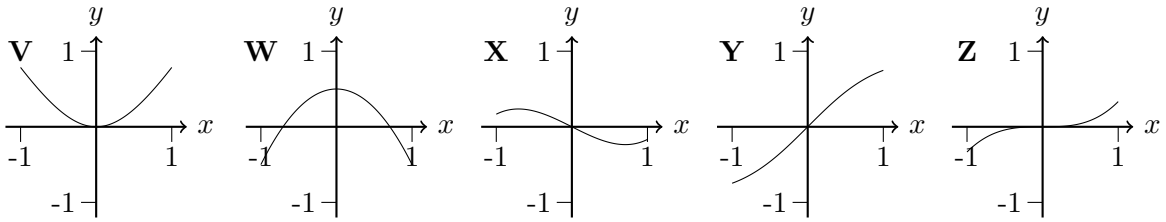
a. $2f(x)+1$
b. $f(2x-1)$
c. $\frac{1}{2}f(x)+4$
d. $\frac{1}{2}f(2x)+4$
e. $-2f(x)+1$
f. $-f(2x+1)$
g. $-\frac{1}{2}f(x)+4$
h. $-\frac{1}{2}f(2x)+4$

11. For each graph in the top row, identify its antiderivative from among the graphs on the bottom row.

Functions:



Antiderivatives:



- a:
- b:
- c:
- d:

_____ **12.** What are two intervals in which the function $f(x) = x^4 - 3x^2 + x + 2$ has a zero?

a. $(-1, 0)$ and $(0, 1)$
b. $(-2, -1)$ and $(-1, 0)$
c. $(-2, -1)$ and $(1, 2)$
d. $(0, 1)$ and $(1, 2)$
e. $(-2, -1)$ and $(2, 3)$
f. $(-1, 0)$ and $(2, 3)$
g. $(0, 1)$ and $(2, 3)$
h. $(1, 2)$ and $(2, 3)$

13. Suppose $f(x) = \frac{1}{x}$. Using the definition of the derivative, find $f'(2)$. (You will receive NO credit for finding the derivative using a different method.)

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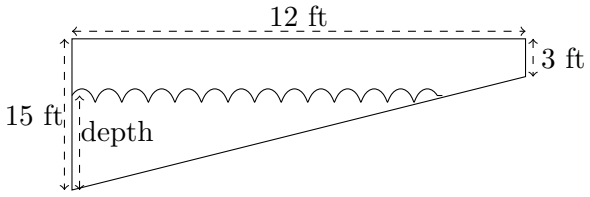
14. A swimming pool is 12 feet long, 10 feet wide, and has a depth that changes from 3 feet deep at the shallow end to 15 feet deep at the deep end. The pool is filling at a rate of 800 cubic feet per minute. At what speed is the depth of the water at the deepest point rising when the water is 5 feet deep.

This picture shows the swimming pool from the side (NOT TO SCALE). (Remember that it also extends 10 feet "into the page".)



15. Find $\frac{d}{dx} \sin e^{\tan x^2}$.

Solutions:

1. b
2. c
3. a
4. d
5. g
6. e
7. f
8. h
9. h
10. g
11. a:Y b:X c:Z d:V
12. b
13. $\lim_{h\to 0} \frac{\frac{1}{2+h}-\frac{1}{2}}{h} = \lim_{h\to 0} \frac{\frac{2-(2+h)}{2(2+h)}}{h} = \lim_{h\to 0} \frac{-h}{2(2+h)h} = \lim_{h\to 0} \frac{-1}{2(2+h)} = \frac{-1}{4}$
14. 16 ft/minute
15. $2x(\sec^2 x^2)e^{\tan x^2} \cos e^{\tan x^2}$