Math 103 Spring 2008   NAME_____

Final Exam

May 9, 2008   Student ID #_____

Circle the name of your professor

GIBNEY   RIMMER

Work all problems in the space provided. You may use the back of each sheet for additional space. Please indicate when you do so.

There are 13 multiple choice questions and 3 free-response partial credit questions. Please write legibly so that the proper credit may be given. You must show all work, an unsupported answer will receive little or no credit.

Transfer your answers (except for # 16) here.

5 pts. each

	Answers
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

	Answers
14 (a)	
14 (b)	
14 (c)	
15 (a)	
15 (b)	
15 (c)	
16	Score for 16:
SCORE	/100

1. For what values of x does the function $f(x) = x^3 - 9x^2 - 120x + 6$ have a local minimum?

(a) 10   (b) 4   (c) 3   (d) −4   (e) −10

2. Evaluate $\displaystyle\int_{0}^{\pi/4} \sin x\,dx + \int_{-\pi/4}^{0} \cos x\,dx.$

(a) $-\sqrt{2}$ (b) -1 (c) 0 (d) 1 (e) $\sqrt{2}$

3. Let $f(x) = \begin{cases} ax^3 - 6x, & \text{if } x \leq 1 \\ bx^2 + 4, & \text{if } x > 1 \end{cases}$

If the $f(x)$ is differentiable, then $a =$

(a) 0　　　　(b) 1　　　　(c) −14　　　(d) −24　　　(e) 26

4. Find a positive value c, for x, that satisfies the conclusion of the Mean Value Theorem for $f(x) = 3x^2 - 5x + 1$ on the interval $[2,5]$.

(a) 1   (b) $\dfrac{13}{6}$ (c) $\dfrac{11}{6}$ (d) $\dfrac{23}{6}$ (e) $\dfrac{7}{2}$

5. Given $f(x) = 2x^2 - 7x - 10$, find the absolute minimum of $f(x)$ on $[-1, 3]$.

(a) -1 (b) $\dfrac{-129}{8}$ (c) -13 (d) $\dfrac{7}{4}$ (e) -20

6. Find the equation of the tangent line to $9x^2 + 16y^2 = 52$ through $(2,-1)$.

(a) $-9x + 8y - 26 = 0$ (b) $9x - 8y - 26 = 0$ (c) $9x - 8y - 106 = 0$

(d) $8x + 9y - 17 = 0$ (e) $9x + 16y - 2 = 0$

7. Let $F(x) = \displaystyle\int_0^{2x} \sin^2 t\, dt$. Find $F'\left(\dfrac{\pi}{6}\right)$.

(a) $\dfrac{-1}{2}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{3}{2}$ (e) $\dfrac{3}{4}$

8. Given f and g are differentiable functions and

$f(a) = -4 \quad g(a) = c \quad f(c) = 15 \quad g(c) = 10$

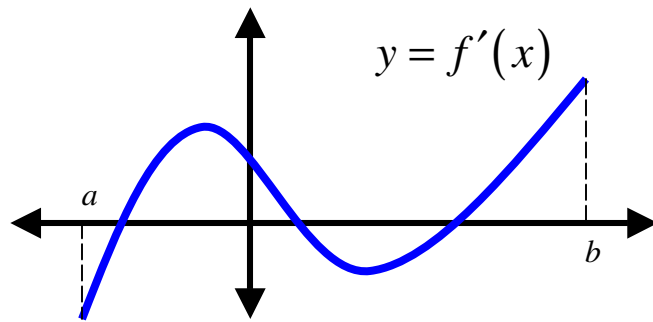$f'(a) = 8 \quad g'(a) = b \quad f'(c) = 6 \quad g'(c) = 5$

If $h(x) = f(g(x))$, find $h'(a)$

(a) $6b$ (b) $8b$ (c) $-4b$ (d) 80 (e) $15b$

9. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

(a) $\dfrac{14}{3}$ (b) $\dfrac{16}{3}$ (c) $\dfrac{28}{3}$ (d) $\dfrac{32}{3}$ (e) 8π

10. The graph of f', the derivative of f, is shown in the figure below. Which of the following describes all relative extrema of f on the open interval (a,b)?

$$y = f'(x)$$

(a) One local maximum and two local minima
(b) Two local maxima and one local minimum
(c) Three local maxima and one local minimum
(d) One local maximum and three local minima
(e) Three local maxima and two local minima

11. Let f be a continuous function on the closed interval $[-3,6]$. If $f(-3)=-1$ and $f(6)=3$, then the Intermediate Value Theorem guarantees that

(a) $f(0)=0$

(b) $f'(c)=\dfrac{4}{9}$ for at least one c between -3 and 6

(c) $-1 \le f(x) \le 3$ for all x between -3 and 6

(d) $f(c)=1$ for at least one c between -3 and 6

(e) $f(c)=0$ for at least one c between -1 and 3

12. Evaluate $\displaystyle\int_{-1}^{3} \sqrt{3+2x}\ dx.$

(a) $\dfrac{8}{3}$ (b) $\dfrac{7}{3}$ (c) $\dfrac{16}{3}$ (d) $\dfrac{26}{3}$ (e) $\dfrac{52}{3}$

13. The sale price an item is $p = 800 - 35x$ dollars and the total manufacturing cost is $2x^3 - 140x^2 + 2600x + 10000$ dollars, where x is the number of items. What number of items should be manufactured in order to maximize the manufacturer's total profit?

(a) 35   (b) 25   (c) 10   (d) 15   (e) 20

14. An object moves along a horizontal line with velocity $v(t) = t^2 - 8t + 7$, $\left(\text{measured in } {}^{m}\!/_{\text{sec.}}\right)$

(a) Write a polynomial expression for the position of the particle at any time $t \geq 0$ if the initial position of the particle is at 4 m.

(b) At what time(s) is the particle changing direction?

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 4$.

15. Water is draining at the rate of 48π $\frac{ft.^3}{sec.}$ from the vertex at the bottom of a conical tank whose diameter at its base is 40 feet and whose height is 60 feet.

(a) Find an expression for the volume of water in the tank in terms of its radius at the surface of the water.

(b) At what rate is the radius of the water in the tank shrinking when the radius is 16 feet?

(c) How fast is the height of water in the tank dropping at the instant that the radius is 16 feet?

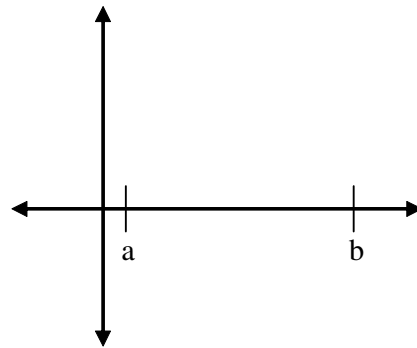16. (a) Sketch the graph of a function that is **increasing** at an **increasing** rate on the interval (a,b).
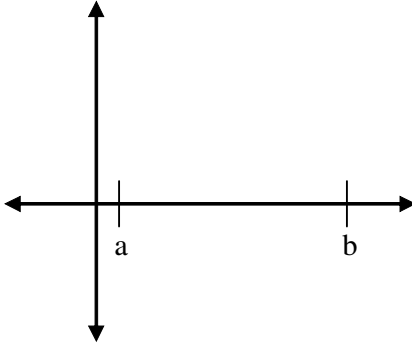
(b) Sketch the graph of a function that is **increasing** at a **decreasing** rate on the interval (a,b).
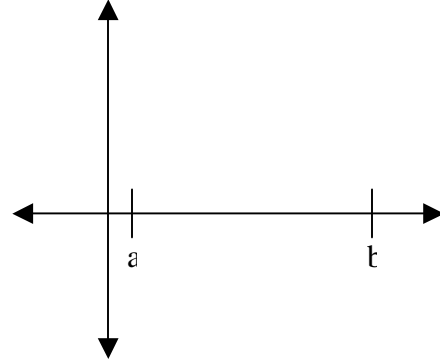
(c) Sketch the graph of a function that is **decreasing** at an **increasing** rate on the interval (a,b).
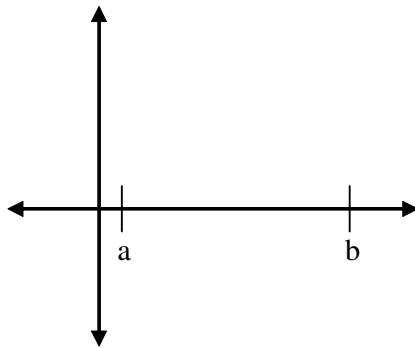
(d) Sketch the graph of a function that is **decreasing** at a **decreasing** rate on the interval (a,b).

(e) Sketch the graph of a function that is **decreasing** at a **constant** rate on the interval (a,b).