LAST NAME:________________ **NAME:**________________ PENN ID # :________________

Please bubble-in the required information.

INSTRUCTOR'S NAME	T.A.'s NAME
SS Stéphane SABOURAU	MT Michael THOMPSON
DS David SANTOS	MY Meesue YOO

Instructions: *You have 120 minutes to complete this exam.* **Do not detach this sheet from the body of the test.** *This is a multiple-choice test. Please show all your work. Answers with no supporting work will be given no points. If you change an answer, please either erase or cross out the answer you do not want considered; questions with more than one answer will be marked wrong. Bubble-in your choice both in the body of the test and in the grid below.*

Questions 1—11	**Questions 12–22**
1. A B C D E F	12. A B C D E F
2. A B C D E F	13. A B C D E F
3. A B C D E F	14. A B C D E F
4. A B C D E F	15. A B C D E F
5. A B C D E F	16. A B C D E F
6. A B C D E F	17. A B C D E F
7. A B C D E F	18. A B C D E F
8. A B C D E F	19. A B C D E F
9. A B C D E F	20. A B C D E F
10. A B C D E F	21. A B C D E F
11. A B C D E F	22. A B C D E F

1. The curve $y = x^2 + 2x - 1$ undergoes the following successive transformations:

   (a) a translation one unit to the left,

   (b) a reflection about the y-axis,

   (c) a reflection about the x-axis.

   What is the equation of the resulting curve?

   A $y = -x^2 - 4x - 2$ D $y = -x^2 + x - 3$
   B $y = x^2 + x + 1$ E $y = -x^2 - 3x - 5$
   C $y = -x^2 + 4x - 2$ F $y = -x^2 + 2$

2. If the function f given by

$$f(x) = \begin{cases} \dfrac{\sin ax}{x} & \text{if } x < 0 \\ \dfrac{x-1}{x^2-1} + b & \text{if } 0 \le x < 1 \\ x + 1 & \text{if } x \ge 1 \end{cases}$$

   is everywhere continuous, find (a, b).

   A $(a, b) = (2, \frac{3}{2})$ D $(a, b) = (1, 2)$

   B $(a, b) = (-1, 1)$ E $(a, b) = (2, 2)$

   C $(a, b) = (2, 1)$ F $(a, b) = (\frac{5}{2}, \frac{3}{2})$

3. Figures 1 and 2 show two functions f and g.

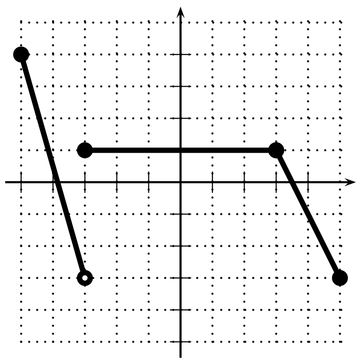

Figure 1: Problem 3. The function f.

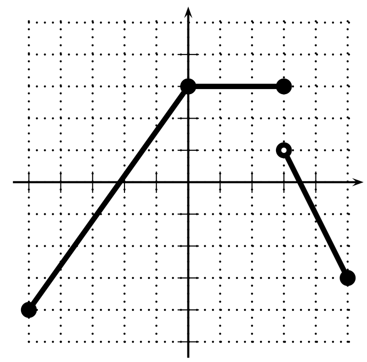

Figure 2: Problem 3. The function g.

Determine $(fg)'(-1) + (f \circ g)'(4)$.

(A) 0 (B) -1 (C) 1 (D) $\frac{3}{2}$ (E) 4 (F) $\frac{7}{5}$

4. If $3x^2 + 2xy + y^2 = 2$, find the value of y' at $x = 1$.

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4 (F) undefined

5. Find all the x-values of the points on the graph of $f(x) = \dfrac{4x-1}{2x+1}$ where the tangent line to f is parallel to $y = 6x - 3$.

(A) $x \in \{0, 1\}$ (B) $x \in \{-1, 1\}$ (C) $x \in \{0, -1\}$ (D) $x \in \{-2, -1\}$ (E) $x \in \{-1, 2\}$ (F) $x \in \{1, 2\}$

6. The curve

$$y = x^3 + 3x^2 + ax + b$$

has one inflection point. The tangent line at this inflection point is $y = 3x + 4$. Find the constants a and b.

(A) $(a, b) = (6, 5)$ (D) $(a, b) = (5, 6)$

(B) $(a, b) = (3, 1)$ (E) $(a, b) = (-1, 3)$

(C) $(a, b) = (-6, 5)$ (F) $(a, b) = (6, 4)$

7. A right circular cylinder is inscribed in a cone with height 1 meter and base radius 1 meter. What is the largest possible volume of such a cylinder?

(A) $\frac{8}{27}\pi$ (B) $\frac{2}{3}\pi$ (C) $\frac{9}{8}\pi$ (D) $\frac{4}{27}\pi$ (E) $\frac{4}{81}\pi$ (F) $\frac{2}{27}\pi$

8. Let f be a function such that $f'(1) = 0$ and $f''(x) = x^4 - a^2x^2 - 2a - 1$. Which of the following conditions always guarantees that f has a local minimum at $x = 1$?

(A) $a < -2$ (B) $-2 < a < 0$ (C) $a > 0$ (D) $-2 < a < 2$ (E) $-2 < a < 1$ (F) $a < 2$

9. Find the absolute minimum of $f(x) = x^3 - 3x^2 - 4$ when $-2 \le x \le 3$.

(A) 2 (B) 0 (C) -4 (D) -8 (E) -24 (F) -48

10. The graph of the *derivative* f' of a certain function f defined on $[-3, 2]$ is given in figure 3 below.

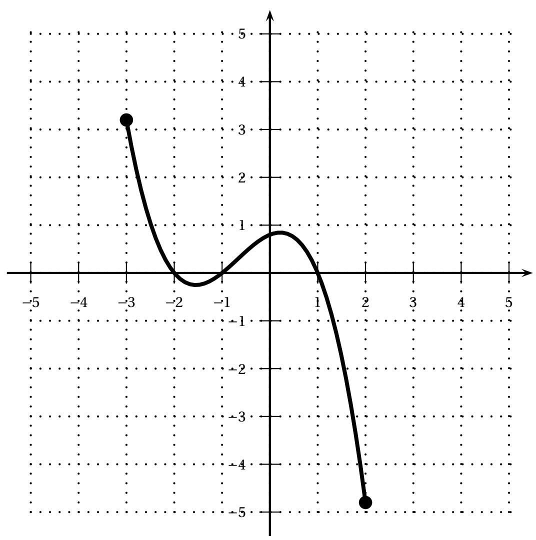

Figure 3: Problem 10.

Find the x-coordinate of the local minimum of f. You must explain your choice.

(A) $x = -3$ (B) $x = -2$ (C) $x = -1$ (D) $x = 0$ (E) $x = 1$ (F) $x = 2$

11. Consider a function f whose *derivative* is $f'(x) = \dfrac{\sqrt{x}}{1+x}$. Find the x-coordinate of the inflection point of the curve $y = f(x)$.

(A) $x = -2$ (B) $x = -1$ (C) $x = 0$ (D) $x = 1$ (E) $x = 2$ (F) $x = \frac{1}{2}$

12. If $f(x) = x^3 - x^2$, then according to the Mean Value Theorem, there is a point c in the interval $(0,1)$ such that
$$f'(c) = \frac{f(1) - f(0)}{1 - 0}.$$
Find c.

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$ (E) $\frac{3}{2}$ (F) $\frac{1}{3}$

13. Determine $\dfrac{\mathrm{d}}{\mathrm{d}x}\displaystyle\int_x^{x^2} \sqrt{1+t^2}\,\mathrm{d}t$.

(A) $2x\sqrt{1+x^4} - \sqrt{1+x^2}$ (D) $\sqrt{1+x^4} - \sqrt{1+x^2}$

(B) $(x^2 - x)\sqrt{1+x^2}$ (E) $x^4 - 2x^2 + 1$

(C) $x^2\sqrt{1+x^2} - x\sqrt{1+x^2}$ (F) $\frac{2}{3}(1+x^4)^{3/2} - \frac{2}{3}(1+x^2)^{3/2}$

14. Evaluate $\displaystyle\int_0^{\pi} \frac{\sin x}{\sqrt{3+\cos x}}\,dx$.

(A) $2 - 2\sqrt{2}$ (B) $4 - 2\sqrt{3}$ (C) 2 (D) $4 + 2\sqrt{2}$ (E) $4 - 2\sqrt{2}$ (F) 0

15. If f is continuous and $\displaystyle\int_0^4 f(x)\,dx = 9$, find $\displaystyle\int_0^2 x f(x^2)\,dx$.

(A) 18 (B) $\frac{9}{2}$ (C) $\frac{9}{4}$ (D) -9 (E) 9 (F) 36

16. The graph of a function f defined on $[-3,4]$ is given in figure 4 below. It is composed of a semi-circle and lines.

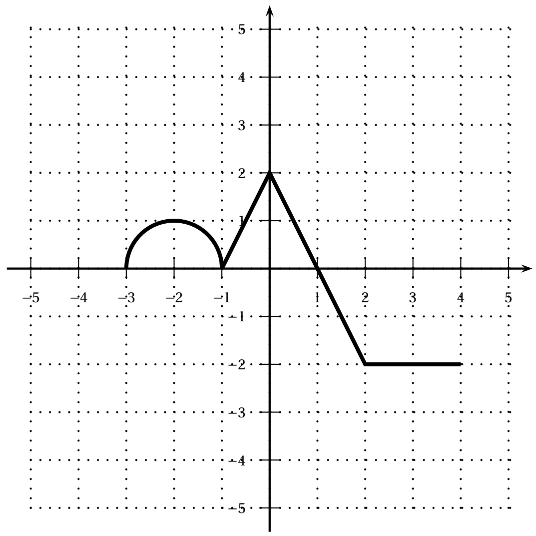

Figure 4: Problems 16 and 17.

If $g(x) = \int_{-3}^{x} f(t)\,\mathrm{d}t$, find $g(2)$.

(A) $\frac{\pi}{2}+2$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{2}+1$ (D) $\frac{\pi}{2}+\frac{3}{2}$ (E) $\frac{\pi}{2}-3$ (F) $\pi+1$

17. Where does g above attain its absolute maximum value?

(A) $x=-3$ (B) $x=-1$ (C) $x=0$ (D) $x=1$ (E) $x=2$ (F) $x=4$

18. Find $\int_{1}^{3} (|x+1|+|x-2|)\,\mathrm{d}x$.

(A) 9 (B) 8 (C) 3 (D) 4 (E) 5 (F) 7

19. A function f satisfies $f(x+h)-f(x) = (2x-1)h+h^2$ and $f(0)=1$. Determine $f(2)$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5

20. Find the area of the region bounded by the the curves $x = 2y^2$ and $x+y=1$.

(A) $\frac{2}{3}$ (B) $\frac{9}{8}$ (C) $\frac{3}{8}$ (D) $\frac{5}{6}$ (E) $\frac{3}{4}$ (F) $\frac{7}{24}$

21. Find the volume of the solid obtained by rotating the region bounded by the the curves $y = 1 + x^2$ and $y = 9 - x^2$ about the x-axis.

(A) $\frac{640}{3}\pi$ (B) $\frac{64}{3}\pi$ (C) $\frac{320}{3}\pi$ (D) $\frac{440}{3}\pi$ (E) $\frac{220}{3}\pi$ (F) $\frac{44}{3}\pi$

22. Figure 5 shows the region $\mathscr{R}$ enclosed by two functions f and g. It is known that

$$0 \leq g(x) \leq f(x) \quad \text{for } x \in [1;4]; \qquad f(1) = g(1) = 2; \qquad f(4) = g(4) = 4,$$

and that

$$\int_1^4 (f(x))^2 \, dx = A; \qquad \int_1^4 f(x) \, dx = a;$$

$$\int_1^4 (g(x))^2 \, dx = B; \qquad \int_1^4 g(x) \, dx = b;$$

$$\int_1^4 f(x)g(x) \, dx = \Gamma$$

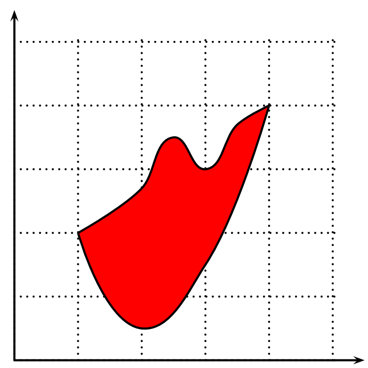

Figure 5: Problem 22.

The volume generated when $\mathscr{R}$ is revolved about the line $y = -1$ is

(A) $\pi(A - B + 2a - 2b)$

(B) $\pi(A + B)$

(C) $2\pi(A - B + a - b)$

(D) $\pi(A - 2\Gamma + B)$

(E) $\pi(A - B + a - b)$

(F) $2\pi(A^2 - B^2 + a^2 - b^2)$

103 Answer Key for Version A: Sabourau and Santos

1. C
2. F
3. F
4. F
5. C
6. A
7. D
8. B
9. E
10. C
11. D
12. C
13. A
14. E
15. B
16. C
17. D
18. F
19. D
20. B
21. A
22. A