Name _____

PennCard ID _____

There are 12 questions on the exam with a total of 200 possible points. Some questions count 15 points and some count 20 points. The value of each question is given after the question number.

No calculators are allowed on the exam, but you may use the pages of notes provided with the exam.

Be sure to show your work in the space provided and write your answers in the boxes provided.

Good luck!

Do not write below this line

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TOTAL _____

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1. (20 points) Compute the following limits.

a) $\lim\limits_{x \to 1} \dfrac{3^x - 3}{x^2 - 1} =$

b) $\lim\limits_{x \to 0} \dfrac{e^{2x} - 1}{\sin(5x)} =$

2. (15 points) Find the equation of the tangent line to the curve $xy^3 - x^2 y = 6$ at the point (3,2).

The equation of the tangent line is _____

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3. (20 points)
Consider the area in the plane bounded by the the lines, x=0, y=5, and the curve, $y = e^x$.

This area can calculated as $\int_a^b f(x)dx$ for some function f(x) and real numbers a and b.

   i)   What is f(x) ?

   ii)   What are a and b ?

   iii)   Calculate the area.

f(x) = _____   a = _____   b= _____   Area = _____

4. (20 points) Let $f(x) = x^2 e^{-x} + 3$

   a) What are global <u>maximum and minimum values of f(x)</u> for $x \geq 0$.

   b) What is $\lim_{x \to \infty} f(x)$?

Global maximum = _____   Global minimum = _____

$\lim_{x \to \infty} f(x)$ = _____

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5. (20 points) Evaluate the following two integrals:

a) $\displaystyle\int_{0}^{\frac{\pi}{2}} \frac{\cos(x)}{2+\sin(x)}\,dx$

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│                                            │
│   ANSWER = _____ │
│                                            │
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b) $\displaystyle\int_{1}^{2} \frac{5+\sqrt{3t}}{t}\,dt$

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│   ANSWER = _____ │
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6. (15 points) Suppose that $\displaystyle\int_{0}^{x^2} f(x)\,dx = \sqrt{x^2+1}-1$ What is the value of f(2) ?

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│                        │
│   f(2) = _____    │
│                        │
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7. (15 points) A certain solid has as its base the quarter circle, $x^2 + y^2 = 4$ with $x \geq 0$ and $y \geq 0$..
The cross sections perpendicular to the x axis are squares. Compute the volume of this solid.
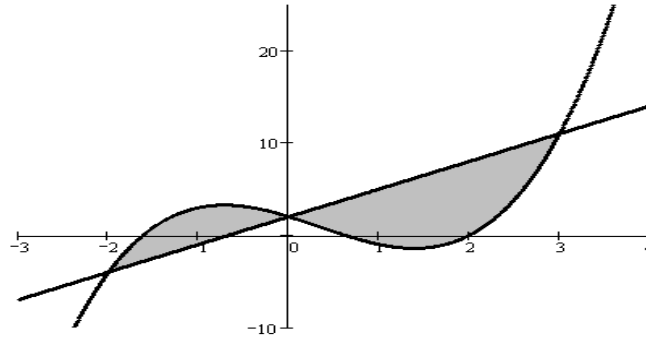
VOLUME = _____

8. (20 points) A rectangular sheet of paper is to contain 72 square inches of printed matter with 2 inch margins at the top and bottom and 1 inch margins on each side. What <u>dimensions for the sheet</u> will use the least paper?

LENGTH = _____   WIDTH = _____

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9. (15 points)

The area between the curves $y = x^3 - x^2 - 3x + 2$ *and* $y = 3x + 2$ is the shaded area in the picture above. Which of the integrals below represents this area. Circle all correct answers.

a) $\displaystyle\int_{-2}^{3} ((x^3 - x^2 - 3x + 2) - (3x + 2))dx$

b) $\displaystyle\int_{-2}^{3} |\, x^3 - x^2 - 6x \,|\, dx$

c) $\displaystyle\int_{-2}^{3} ((3x + 2) - (x^3 - x^2 - 3x + 2))dx$

d) $\displaystyle\int_{-2}^{0} ((x^3 - x^2 - 3x + 2) - (3x + 2))dx + \int_{0}^{3} ((3x + 2) - (x^3 - x^2 - 3x + 2))dx$

e) $\displaystyle\int_{-2}^{0} ((3x + 2) - (x^3 - x^2 - 3x + 2))dx + \int_{0}^{3} ((x^3 - x^2 - 3x + 2) - (3x + 2))dx +$

f) $\displaystyle\int_{-2}^{0} (x^3 - x^2 - 6x)\, dx - \int_{0}^{3} (x^3 - x^2 - 6x)\, dx$

10. (20 points) Water is being pumped into an inverted conical tank. The tank has height of 12 feet and the radius at the top is 4 feet. If the water level is rising at a rate of 1.5 feet per minute when the height of the water is 2 feet, find the rate at which water is being pumped into the tank in cubic feet per minute. (Note: The volume of a cone of height h and radius r is $\dfrac{\pi r^2 h}{3}$. You may leave your answer in terms of π.)
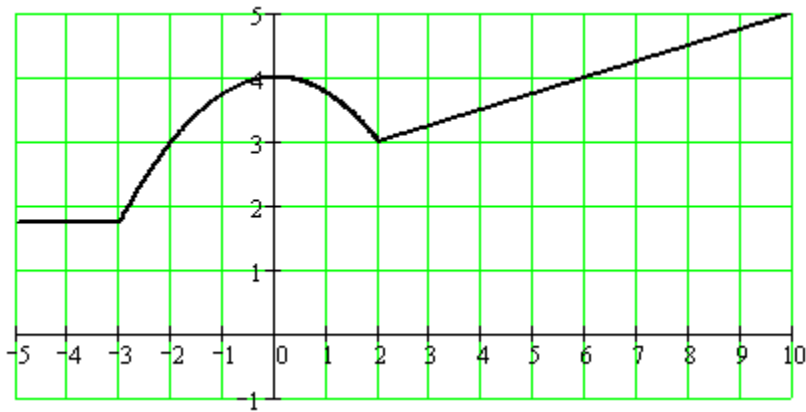
Water is being pumped into the tank at the rate of _____ cubic feet per minute.
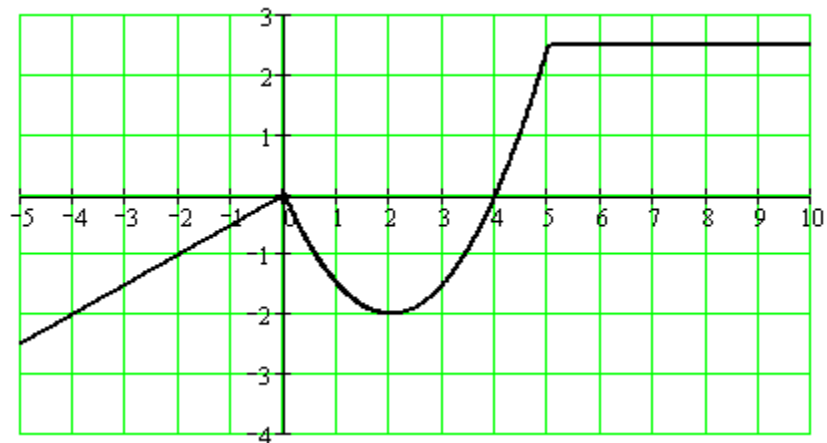
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11. (20 points) The graph below is the graph of f(x).
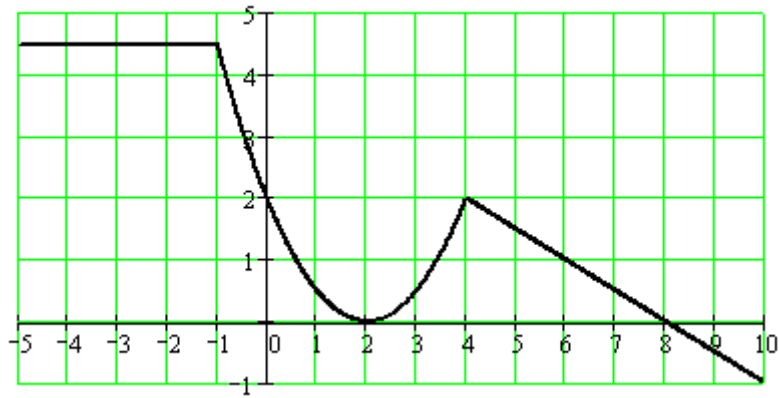
The two graphs below are the graphs of $af(bx + c) + d$ for appropriate choices of real numbers a, b, c and d. In each case determine the numbers, a, b, c and d. (Note that the scales of the graphs below may difer from the scale of the original function.)
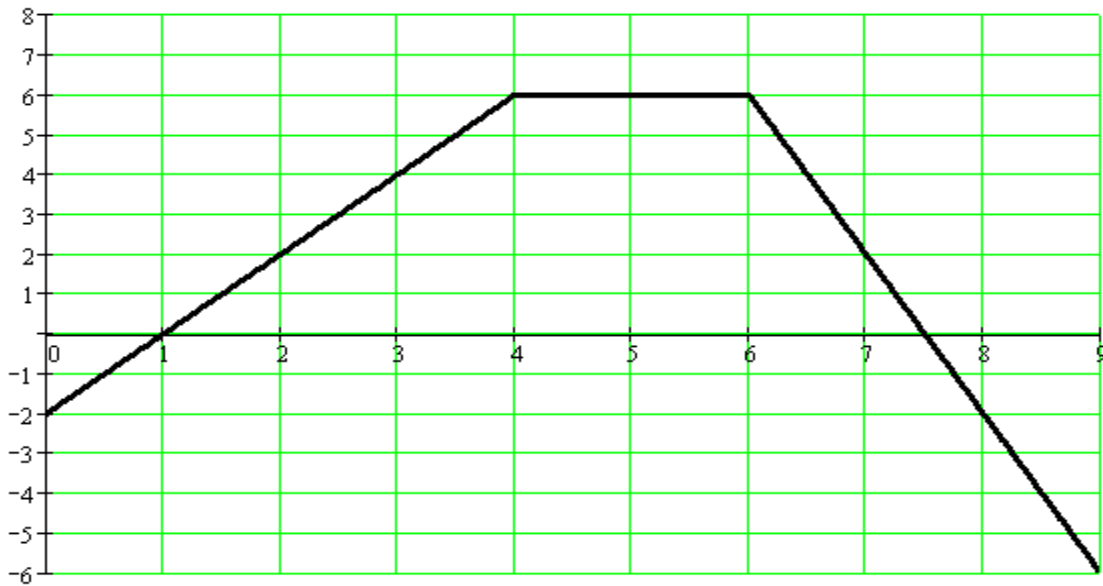

a=_____ b=_____ c=_____ d=_____

a=_____ b=_____ c=_____ d=_____

12. (20 points) The graph below is the graph of $f'(x)$ for some function f(x).

You are given that f(0) = 5. Fill in the following table:

X	0	1	2	3	4	5	6	7	8	9
f(x)	5									

i)   For what value of x does f(x) have a global minimum? _____

ii) For what value of x does f(x) have a global maximum? _____

iii) For what values of x is f(x) concave up?_____

iv) For what values of x is f(x) increasing? _____