Final Exam for Math 103, Fall 2004

1. Evaluate the integral

$$\int_{-3}^{0} (1 + \sqrt{9 - x^2})\, dx$$

by interpreting it as the area of a region involving basic geometric figures.

(a) $\dfrac{9\pi}{4}$

(b) $3 + 9\pi$

(c) $3 + \dfrac{9\pi}{4}$

(d) $9 + 3\pi$

(e) $\dfrac{3\pi}{4}$

(f) $9 + \dfrac{3\pi}{4}$

---

2. Evaluate the integral

$$\int_{0}^{3} \frac{x + 4}{x^2 + 8x + 1}\, dx$$

.

(a) 0 (b) $\dfrac{1}{2}\ln 34$ (c) $\ln 49$ (d) $\dfrac{1}{34}$ (e) $\dfrac{1}{49}$ (f) $\ln 34 - \dfrac{1}{2}$

---

3. What is the equation of the line tangent to the graph of $y^3 + 3x^2 y^2 + 2x^3 = 4$ at the point $(1, -1)$?

(a) $y = -1$

(b) $y = x - 2$

(c) $y = 2x - 3$

(d) $y = 3x - 4$

(e) $y = 4x - 5$

(f) $y = 5x - 6$

---

4. A particle moves in such a way that its distance from the origin at time t is given by $x(t) = 2\sqrt{t^2 + 4}$. If $v(t)$ is the velocity of the particle at time t, what is $\lim\limits_{t \to \infty} v(t)$?
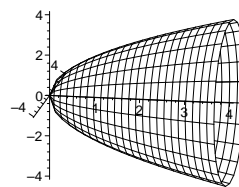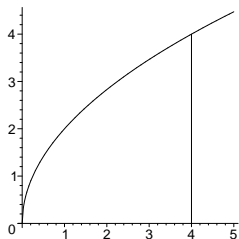
(a) 2 (b) $1/2$ (c) $1/4$ (d) $1/\sqrt{2}$ (e) 0 (f) ∞

---

5. What are the global maximum and minimum *values* of the function $f(x) = \dfrac{x}{1 + x^2}$?

(a) 2 and -2

(b) 1 and -1

(c) $1/2$ and $-1/2$

(d) 2 and 0

(e) 4 and -4

(f) ∞ and $-\infty$

---

6. The region bounded by the curve $y = 2\sqrt{x}$, the x-axis, and the line $x = 4$ is revolved about the x-axis, creating a solid. What is the volume of the solid?

(a) 32π (b) $\dfrac{32\pi}{3}$ (c) 64π (d) $\dfrac{64\pi}{3}$ (e) 72π (f) $\dfrac{72\pi}{3}$

---

7. A stock market analyst sold a monthly newsletter to 320 subscribers at a price of $10 each. She discovered that for each $0.25 increase in the monthly price of the newsletter, she would lose 2 subscriptions. If she sets the price of the newsletters to bring in the greatest total monthly income, what will that income be?

(a) $3200 (b) $4400 (c) $5000 (d) $5800 (e) $6500 (f) $7200

---

8. What is $\lim\limits_{x \to 1} \dfrac{x^2 + 2x - 3}{x - 1}$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4 (f) does not exist.

---

9. Water is draining from a conical tank at the rate of 18 cubic feet per minute. The tank has a height of 10 feet and the radius at the top is 5 feet. How fast (in feet per minute) is the water level changing when the depth is 6 feet? (Note: The volume of a cone of radius r and height h is $\pi r^2 h/3$.)

(a) $\dfrac{1}{\pi}$ (b) $\dfrac{2}{\pi}$ (c) $\dfrac{3}{\pi}$ (d) $-\dfrac{1}{\pi}$ (e) $-\dfrac{2}{\pi}$ (f) $-\dfrac{3}{\pi}$
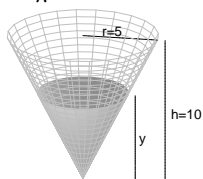
10. Suppose

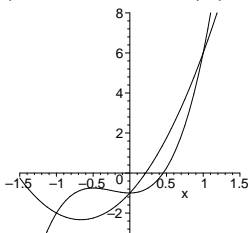$$\int_0^x f(x)\,dx + 2\sin x = 4x.$$

What is the value of $f(\pi)$?

(a) 2   (b) 4   (c) 6   (d) 2π (e) 4π (f) 6π

---

11. Compute $\displaystyle\int_1^2 3^x\,dx$.

(a) $\dfrac{7}{2}$ (b) e^3 (c) $3\ln 6$ (d) $\dfrac{6}{\ln 3}$ (e) 6   (f) $\dfrac{\ln 3}{6}$

---

12. What is the *total* area enclosed between the graphs of $y = 4x^3 + 3x^2 - 1$ and $y = 3x^2 + 4x - 1$?

(a) 1   (b) 2   (c) 3   (d)4   (e) 5   (f) 6

Answers

1. C
2. B
3. E
4. A
5. C
6. A
7. C
8. E
9. E
10. C
11. D
12. B