Math 103 **Fall 2003**
Final Examination **December 18, 2003**

Name _____

Student ID number _____

Lecture section (circle): DeTurck (001) Porter (002) Clee (003)

Section _____ T.A. _____ Recitation time _____

There are fifteen "multiple choice" questions on this examination. No calculators are allowed, but you may use the pages of notes that are provided with the exam.

Be sure to show your work in the space provided, and indicate your answers clearly. For each question, circle the correct answer or answers as well as showing your work. We reserve the right to adjust raw multiple choice scores based on the amount and quality of supporting work.

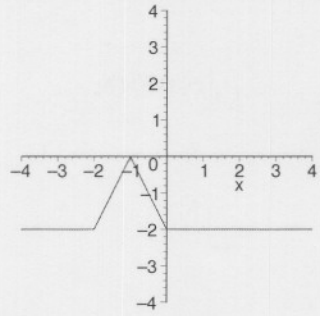
Note that two of the problems (#4 and #14) may have more than one correct answer (and you should choose *all* correct answers for these. Transfer your answers to this page.

Good luck!

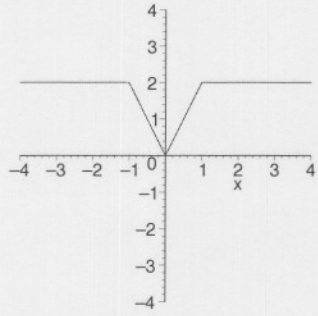
1. _____	**6.** _____	**11.** _____
2. _____	**7.** _____	**12.** _____
3. _____	**8.** _____	**13.** _____
4. _____	**9.** _____	**14.** _____
5. _____	**10.** _____	**15.** _____

Name _____

1. Here is the graph of a certain function, $y = f(x)$:



Here is a graph of a related function:



Which of the following is the equation of this graph?

(a) $y = f(x+1)$ (b) $y = f(x+1) - 1$ (c) $y = -f(x+1)$
(d) $y = -f(x-1)$ (e) $y = 1 - f(x-1)$ (f) $y = f(x-1) - 1$

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2. If the function defined as

$$f(x) = \begin{cases} e^{x+c} & \text{for } x \leq 0 \\ \dfrac{\sin(3x)}{x} & \text{for } x > 0 \end{cases}$$

is continuous for all x, then what is the value of c?

(a) 0 (b) 1 (c) 3 (d) e^3 (e) e^{-3} (f) $\ln 3$

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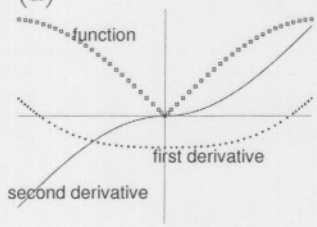
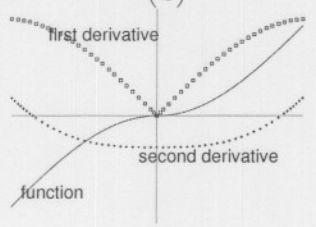
3. The following shows the graph of a function $f(x)$, together with the graph of its first derivative $f'(x)$ and its second derivative $f''(x)$. Each answer shows the same three graphs, but only one is labelled correctly. Which one?

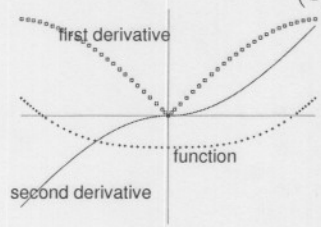
(a)



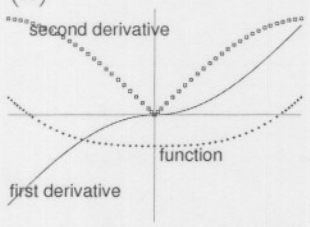
(b)



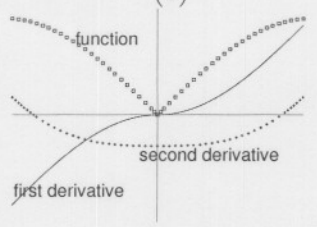
(c)



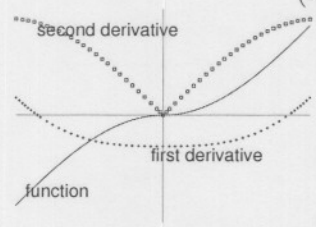
(d)



(e)



(f)



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4. The graph of $y = x^2e^{-4x}$ has (choose *all* that apply):

   (a) a local minimum at $x = 2$
   (b) a local maximum at $x = 2$
   (c) a local minimum at $x = 0$
   (d) a local maximum at $x = 0$
   (e) a local minimum at $x = 1/2$
   (f) a local maximum at $x = 1/2$

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5. $\lim_{x \to \pi/6} \dfrac{2\sin(2x) - \sqrt{3}}{x - \frac{\pi}{6}} =$

(a) 2 (b) -2 (c) $-2\sqrt{3}$ (d) $2\sqrt{3}$ (e) $4\sqrt{3}$ (f) ∞

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6. The point (4,2) is on the graph of $x = y^3 + 3xy - 28$. What is $\dfrac{dy}{dx}$ there?

(a) 0 (b) 1 (c) 7/24 (d) $-5/24$ (e) 7/12 (f) $-5/12$

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7. One of the following lines through the origin is also tangent to the graph of $y = \dfrac{1}{12-x}$. Which one? (*Hint*: Start by finding the equation of the tangent line to the graph at a generic point $(a, \dfrac{1}{12-a})$.)

   (a) $y = x$ (b) $y = x/6$ (c) $y = 12x$ (d) $y = x/12$ (e) $y = x/144$ (f) $y = x/36$

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8. A 12-foot ladder is leaning against a vertical wall. At the moment that the angle between the ladder and the ground (at the bottom of the ladder) is $\pi/6$ radians, the bottom of the ladder is sliding away from the wall at a speed of 3 ft/sec. How fast, in radians per second, is the angle between the ladder and the ground increasing at that moment?

   (a) $\dfrac{1}{\sqrt{3}}$ (b) $\dfrac{\sqrt{3}}{12}$ (c) $\dfrac{12}{5}$ (d) $\dfrac{5}{12}$ (e) $\dfrac{\sqrt{3}}{5}$ (f) $\dfrac{1}{2}$

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9. A cylindrical container is to be constructed using $\$ 600\pi$ worth of material. The sides are to be made of a special material that costs \$2 per square foot, while the material for the top and bottom costs only \$1 per square foot. What is the relationship between the radius r and the height h of the largest (maximum volume) container that can be constructed according to these specifications?

(a) $h = 3r$ (b) $r = 3h$ (c) $h = 2r$ (d) $r = 2h$ (e) $h = r$ (f) $h = 4r$

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10. $\int_1^e \frac{\ln x}{2x}\, dx =$

(a) 1 (b) 1/2 (c) 1/4 (d) $\frac{e^2}{2}$ (e) $e^2/4$ (f) e

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11. Find the total area of the bounded regions between the graph of $y = 6x^2 - 8x - x^3$ and the x-axis.

(a) 0 (b) 2 (c) 4 (d) 6 (e) 8 (f) 10

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12. On which of the following sets is the function $f(x) = \int_0^x e^{t^3-3t}\,dt$ concave down?

(a) $x \in (-1, 1)$ (b) $x \in (-\infty, -1) \cup (1, +\infty)$ (c) $x \in (-\infty, 0)$

(d) $x \in (0, +\infty)$ (e) $x \in (-\sqrt{3}, \sqrt{3})$ (f) $x \in (\ln 2, +\infty)$

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13. A solid is obtained by revolving the region between the lines $x = 1$ and $x = 3$, the x-axis, and the graph of $y = 1/(x + 5)$ around the x-axis. What is its volume?

(a) $\frac{\pi}{24}$ (b) $\frac{\pi}{8}$ (c) 2π (d) $\frac{3\pi}{8}$ (e) $\frac{3\pi}{4}$ (f) $\frac{4\pi}{3}$

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14. Let $f(x)$ be a function with the properties that $f'(x) < 0$ for all x, and that $f(0) = 8$, $f(1) = 5$, $f(3) = 2$ and $f(4) = 1$. Which of the following *must* be true? Choose *all* that apply. (*Hint*: Draw a picture!)

(a) $\int_0^4 f(x)\,dx > 20$ (b) $\int_0^4 f(x)\,dx < 20$ (c) $\int_0^4 f(x)\,dx < 16$

(d) $\int_0^4 f(x)\,dx > 16$ (e) $\int_0^4 f(x)\,dx > 10$ (f) $\int_0^4 f(x)\,dx > 24$

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15. Suppose $f(x)$ is a continuous function and that $\int_0^9 f(x)\,dx = 7$. Calculate $\int_0^3 xf(x^2)\,dx$.

(a) 7 (b) $\sqrt{7}$ (c) $\frac{7}{2}$ (d) 21 (e) $\frac{21}{2}$ (f) $\frac{\sqrt{7}}{2}$