UNIVERSITY *of* PENNSYLVANIA
MATHEMATICS DEPARTMENT
MATHEMATICS 103–SPRING 2007
FINAL EXAMINATION

Name: ____

Professor (check one): ❑ Crotty (001) ❑ Crotty (601) ❑ Gibney (002)

Instructions:

1. Work all problems in the space provided. You may use the back of each sheet if you need additional space. *An answer, even a correct answer, with no supporting work will receive little or NO credit.*
2. *THIS IS A CALCULUS TEST. Answers, even correct ones, obtained by non-calculus methods will receive little or NO credit.*
3. Transfer all of your answers to the appropriate spaces on this page. Blank or illegible answers are *wrong.*
4. Each short answer is worth 5 points for a total of 60 points with *no* partial credit given.
5. Each free response question is worth 20 points for a total of 40 points; partial credit *may* be given if you provide sufficient work for your method of solution to be evaluated.
6.

Short Answer Questions

Question	**Your Answer**
1	$f'(\frac{\pi}{4}) =$
2	$\int_{\pi}^{2\pi} \cos^2 x \sin x dx =$
3	limit =
4	maximum $f(10)$ = ________________ minimum $f(10)$ = ________________
5	a) x = ___________________________ b) x = ___________________________
6	Area =
7	$\int_3^6 f(x)dx =$
8	$I'(x) =$
9	Equation of tangent line: ___________________________ $y(3.5) \approx$ ____________
10	*True* *False*
11	*True* *False*
12	*True* *False*
13	*True* *False*
14	*True* *False*
15	*True* *False*

Free Response Questions

1	Position of pumping station: x = _____________	2	Horizontal tangents at x = __________________________ Vertical tangents at x = __________________________

Short Answer Questions:

Work each question in the space provided. Short answer questions require NO algebraic simplification.

1. If $f(x) = x^2 \sin x$, find $f'(\frac{\pi}{4})$.

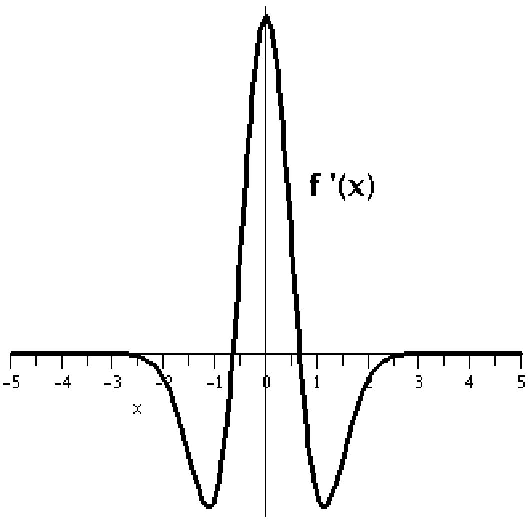
[4 pts]

2. Evaluate $\int_{\pi}^{2\pi} \cos^2 x \sin x dx$.

3. Evaluate $\lim_{x\to\infty} \dfrac{x^2 - 3x + 29034}{7x^2 - 9999x + 2}$

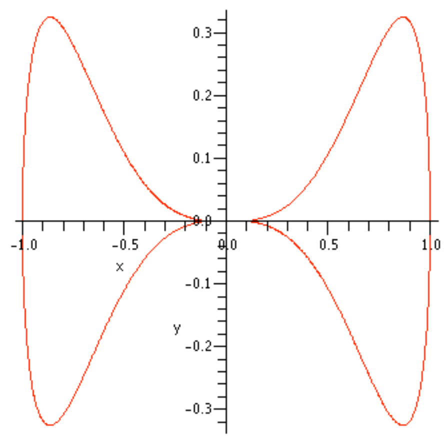
4. Suppose you know that a certain function, $f(x)$ has a derivative, $f'(x)$, which has values in the range $-4 \le f'(x) \le 5$ when x is between 6 and 10. Find the maximum and minimum possible values of $f(10)$ if $f(6) = 17$.

5a. Consider the graph of the *derivative* (***f'(x)***) of some function $f(x)$ as shown below. At what value(s) of x does $f(x)$ have a local maximum [NOTE: if none, write *none* in the proper space on your answer sheet]?



5b. Again using the graph in problem 5, above, at what value(s) of x does the function $f(x)$ whose derivative is shown have a point (or points) of inflection [NOTE: if none, write *none* in the proper space on your answer sheet]?

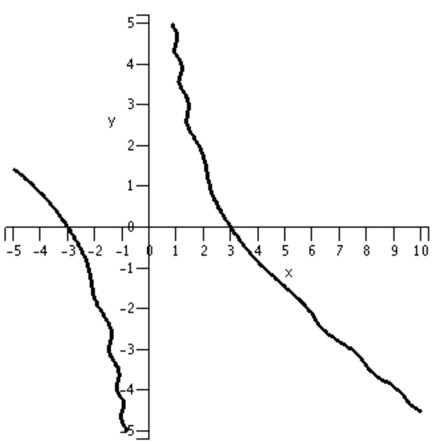
6. Find the area of the region bounded by the curve: $y^2 = x^6(1-x^2)$. (See graph, below) You may find the substitution $x = \sin\theta$ helpful in evaluating the integral.



7. Suppose you know that $\int_0^6 f(x)dx = 10$ and $\int_0^3 f(x)dx = -4$. What is the value of $\int_3^6 f(x)dx = ?$

8. A function $I(x)$ is defined by the integral $I(x) = \int_x^{x^2} \frac{\sin t}{\sqrt{t}}\, dt$. Find the first derivative of $I(x)$.

9. Consider the equation $x^2 + 2xy + \cos(y^2) = 10$. Find the equation of the line tangent to this graph at the point (3, 0). Use your equation to approximate y when $x = 3.5$. Show your calculations here.



Questions 10-15: Decide if each statement is True or False. Circle the correct answer here and circle *True* or *False* on your answer sheet in the appropriate place.

Assume that $f(x)$ and $g(x)$ are differentiable functions defined for all real numbers x.

10. It is possible that $f(x) > 0$ everywhere, $f'(x) > 0$ everywhere and $f''(x) < 0$ everywhere. (TRUE/ FALSE)

11 f can satisfy $f''(x) > 0$ everywhere, $f'(x) < 0$ everywhere and $f(x) > 0$ everywhere. (TRUE/ FALSE)

12. f and g can satisfy $f'(x) > g'(x)$ for all x and $f(x) < g(x)$ for all x. (TRUE/ FALSE)

13. If $f'(x) = g'(x)$ for all x and $f(x_0) = g(x_0)$ for some $x = x_0$, then $f(x) = g(x)$ for all x. (TRUE/ FALSE)

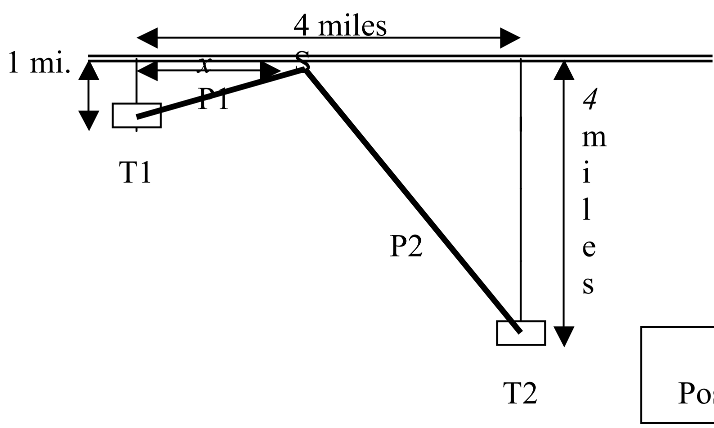
14. If $f''(x) < 0$ everywhere and $f'(x) < 0$ everywhere then $\lim_{x\to+\infty} f(x) = -\infty$. (TRUE/ FALSE)

15. If $f'(x) > 0$ everywhere and $f(x) > 0$ for all x then $\lim_{x\to\infty} f(x) = \infty$. (TRUE/ FALSE)

Free Response Questions:

Show your work here and transfer your results to your answer sheet. Part credit will be given for those parts of these problems properly executed. Each question is worth 10 points.

1. On the same side of a river with straight banks are two towns (T1 and T2) and a pumping station (S) that supplies water to both towns (see diagram below). The pumping station is at the river's edge with pipes extending straight to the distribution points in each town. Where should the pumping station be located (i.e., at what value of *x* in the diagram) to minimize the *total* length of the pipes P1 and P2?



Position of pumping station: $x =$ _______________

2. Given the function $y = (x-1)^{1/2} - \frac{1}{2}(x-1)^{3/2}$, find all points at which the function has a horizontal tangent and all points at which the function has a vertical tangent if any such points exist. NOTE: You MUST show some appropriate calculations—approximating any answers from the graph below will gain you NO POINTS although you may use the graph to check the reasonability of your calculations…

Horizontal tangents at x = ________________________

Vertical tangents at x = __________________________
(if none, write NONE in the space above)