Class of 1880 Exam

(Math competition ONLY for Univ. of Pennsylvania freshmen.)

April 16, 2019

> Solve the problems in the space provided. Show your work and give justification for your answers as completely as possible. If you run out of room for an answer, continue on the back of the page, or on the last 2 pages – Extra Space.

Time available: 2 hours!

Full name: _____

Penn ID: _____

E-mail: _____

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (10 points) Suppose that a house is divided into 27 rooms along a $3 \times 3 \times 3$ grid, and that any two rooms that share a wall are accessible to each other. Show that there is no path starting at the center room that passes through each room exactly once and ends in a room in the corner.

2. (10 points) Prove that among any $n + 1$ positive integers that are less than or equal to $2n$, there must be a pair a and b such that a divides b.

3. (10 points) Bob and Alice play the following game: they have 100 pieces of candy in a bag, and they take turns taking out candy from the bag. On each turn the player must take at least one piece of candy and can take at most eight pieces of candy. The one who takes the last piece of candy from the bag wins the game. Alice goes first. Is there a strategy for her to win the game? Under what circumstances is there a strategy for Bob to win the game, once Alice makes her first move?

4. (10 points) Prove that the sum of the squares of six consecutive integers cannot be a square.

5. (10 points) A triangle has sides of length 13, 14, 15. Find its area.

Extra space

Extra space