Class of 1880 Exam

(Math competition ONLY for UPenn Freshmen.)

April 17, 2018

Solve the problems in the space provided. Show your work and give justification for your answers as complete as possible. If you run out of room for an answer, continue on the back of the page, or on the last 2 pages – Extra Space.

Time available: 2 hours!

Full name: _____

Penn ID: _____

E-mail: _____

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (10 points) We are given four points P, A, B, C in the plane such that $\angle APB = 60$ degrees, the distance from C to B is 3, and the distance from C to A is 2. Moreover, $\angle PAC = \angle PBC = 90$ degrees. Find the distance from P to C.

2. (10 points) A polynomial of degree one in two variables has the form

$$P(x,\, y) = ax + by + c.$$

We are given three distinct points A_1, A_2, A_3 in the real plane. Are the coefficients a, b, c of P determined by the values v_1, v_2, v_3 of P at the points A_1, A_2, A_3? (Note: Your answer may depend on the position of the three points.)

3. (10 points) Find the smallest integer greater than 1 with the property that it is equal to the sum of the cubes of its digits (when written in base 10).

4. (10 points) There are n ($n \geq 3$) given points in the plane such that any three of them are the vertices of a right triangle. Find the largest possible such n.

5. (10 points) A grid with 3 rows and 50 columns is tiled with 75 identical dominoes (each covering two adjacent grid positions). How many ways can this be done such that exactly two of the dominoes are vertical, the rest being horizontal?

Extra space

Extra space