1. Homework 7

Due: In Lecture 10-14

Problem 1. Consider the maps $\phi_t : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$\phi_t(x, y) = (xe^{2t}, ye^{-3t}).$$

(1) Show that $\{\phi_t : t\epsilon\mathbb{R}\}$ is a one-parameter group of linear maps of $\mathbb{R}^2$ to itself.
(2) Find the corresponding vector field V on $\mathbb{R}^2$.
(3) Sketch V and its integral curves.

Problem 2. On $\mathbb{R}^3$, consider the vector fields

$$X = z\frac{\partial}{\partial y} - y\frac{\partial}{\partial z}$$
$$Y = x\frac{\partial}{\partial z} - z\frac{\partial}{\partial x}$$
$$Z = y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}.$$

Show that the flow of $aX + bY + cZ$ is a rotation of $\mathbb{R}^3$ about the line through the origin and the point (a, b, c).

Problem 3. Compute the flow of each of the following vector fields on $\mathbb{R}^2$:

(1) $V = y\frac{\partial}{\partial x} + \frac{\partial}{\partial y}$
(2) $W = x\frac{\partial}{\partial x} + 2y\frac{\partial}{\partial y}$
(3) $X = x\frac{\partial}{\partial x} + -y\frac{\partial}{\partial y}$
(4) $Y = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$