

1. HOMEWORK 6

Due: In Lecture 10-5

Problem 1. Define two smooth atlases on the same topological manifold M^n to be equivalent if their union is a smooth atlas. Prove that this relation is really an equivalence relation.

Problem 2. Let RP^n , real projective n-space, be obtained from the unit sphere $S^n \subset R^{n+1}$ by identifying antipodal pairs of points. Find a smooth structure on RP^n and prove it.

Problem 3. Show how to put a smooth structure on the product $N_1 \times N_2$ of two smooth manifolds.

Problem 4. Let M be a smooth manifold, and let $C^\infty(M)$ denote the set of all smooth real-valued functions $f : M \rightarrow \mathbb{R}$. Show that pointwise addition and pointwise multiplication of smooth functions produce smooth functions.