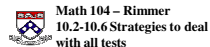
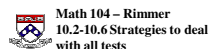


Strategies (which test to use on which series)




- 1) Check at a glance to see if $\lim_{n \rightarrow \infty} a_n \neq 0$.
If this is true, then the series diverges by the **Test for Divergence**.
- 2) Series that we can find whether or not they converge rather quickly:
 - a) **p -series** $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent for $p > 1$ and divergent if $p \leq 1$.
 - b) **geometric series** $\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=1}^{\infty} ar^n$ is convergent for $|r| < 1$ and divergent if $|r| \geq 1$.
- 3) Use the Comparison Test / Limit Comparison Test on series
 - a) that have the form of p -series or geometric series
if a_n is a fraction involving polynomials only or polynomials under radicals
compare this series with a p -series
 - b) Note: make sure that the series has only positive terms to use the comp. tests

12.7 Strategies (which test to use on which series)




- 4) The Alternating Series Test might work on series of the form $\sum_{n=1}^{\infty} (-1)^n b_n$
- 5) The Ratio Test works well on series involving factorials
or constants raised to powers involving n
- 6) The Root Test works well if a_n is of the form $(b_n)^n$
- 7) The Integral Test works well if $\int_k^{\infty} f(x) dx$ is not difficult to evaluate,
where $\sum_{n=k}^{\infty} a_n$ with $a_n = f(n)$ and f is continuous, decreasing, and positive on $[k, \infty)$


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 10.2-10.6 Strategies to deal with all tests


i) $\sum_{n=1}^{\infty} \frac{n}{n^3+2}$	iii) $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$	v) $\sum_{n=1}^{\infty} \pi^{1/n}$	vii) $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$
ii) $\sum_{n=1}^{\infty} \frac{n^2+2}{n^3+3}$	iv) $\sum_{n=1}^{\infty} \frac{\cos(5n)}{1+(1.4)^n}$	vi) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sqrt{n}}{n+1}$	viii) $\sum_{n=2}^{\infty} \frac{n \ln n}{(n-1)^3}$

i) $\sum_{n=1}^{\infty} \frac{n}{n^3+2}$ $b_n = \frac{n}{n^3+2} = \frac{1}{n^2 + \frac{2}{n}}$ $\frac{n}{n^3+2} < \frac{1}{n^2}$ since $n^3 < n^3+2$ $\sum_{n=1}^{\infty} b_n$ converges so $\sum_{n=1}^{\infty} a_n$ converges by the Comp. Test.	ii) $\sum_{n=1}^{\infty} \frac{n^2+2}{n^3+3}$ $b_n = \frac{n^2+2}{n^3+3} = \frac{1}{\frac{n^3+3}{n^2+2}}$ $\lim_{n \rightarrow \infty} \left \frac{a_n}{b_n} \right = \lim_{n \rightarrow \infty} \left \frac{\frac{n^2+2}{n^3+3}}{\frac{1}{n^2+2}} \right = \lim_{n \rightarrow \infty} \left \frac{(n^2+2)^2}{n^3+3} \right = 1$ $\sum_{n=1}^{\infty} b_n$ diverges so $\sum_{n=1}^{\infty} a_n$ diverges by the Limit Comp. Test.	iii) $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$ $\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = \lim_{n \rightarrow \infty} \left \frac{(n+1)^2 e^n}{n^2 e^{n+1}} \right = \lim_{n \rightarrow \infty} \left \frac{(n+1)^2}{n^2} \cdot \frac{1}{e} \right = \frac{1}{e} < 1$ so $\sum_{n=1}^{\infty} a_n$ converges by the Ratio Test.
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 10.2-10.6 Strategies to deal with all tests

i) $\sum_{n=1}^{\infty} \frac{n}{n^3+2}$	iii) $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$	v) $\sum_{n=1}^{\infty} \pi^{1/n}$	vii) $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$
ii) $\sum_{n=1}^{\infty} \frac{n^2+2}{n^3+3}$	iv) $\sum_{n=1}^{\infty} \frac{\cos(5n)}{1+(1.4)^n}$	vi) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sqrt{n}}{n+1}$	viii) $\sum_{n=2}^{\infty} \frac{n \ln n}{(n-1)^3}$

iv) $\sum_{n=1}^{\infty} \frac{\cos(5n)}{1+(1.4)^n}$ terms aren't all positive $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{ \cos(5n) }{1+(1.4)^n}$ now they are $b_n = \frac{1}{(1.4)^n} = \frac{1}{(\frac{7}{5})^n} = \left(\frac{5}{7}\right)^n$ $\sum_{n=1}^{\infty} b_n$ converges geom. series w/ $r=5/7$ $\frac{ \cos(5n) }{1+(1.4)^n} < \frac{1}{(1.4)^n}$ so $\sum_{n=1}^{\infty} a_n$ converges absolutely by the Comp. Test.	v) $\sum_{n=1}^{\infty} \pi^{1/n}$ $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \pi^{1/n} = \pi^0 = 1 \neq 0$ $\Rightarrow \sum_{n=1}^{\infty} a_n$ diverges by the Test For Divergence	vi) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sqrt{n}}{n+1}$ $b_n = \frac{\sqrt{n}}{n+1} > 0$ b_n is decreasing $\lim_{n \rightarrow \infty} b_n = 0$ $\Rightarrow \sum_{n=1}^{\infty} a_n$ converges by the Alternating Series Test
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 10.2-10.6 Strategies to deal
 with all tests

i) $\sum_{n=1}^{\infty} \frac{n}{n^3+2}$ iii) $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$ v) $\sum_{n=1}^{\infty} \pi^{1/n}$ vii) $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$
 ii) $\sum_{n=1}^{\infty} \frac{n^2+2}{n^3+3}$ iv) $\sum_{n=1}^{\infty} \frac{\cos(5n)}{1+(1.4)^n}$ vi) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sqrt{n}}{n+1}$ viii) $\sum_{n=2}^{\infty} \frac{n \ln n}{(n-1)^3}$

vii) $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$ $f(x) = \frac{\ln x}{x^2} > 0$,
 continuous, decreasing

viii) $\sum_{n=2}^{\infty} \frac{n \ln n}{(n-1)^3}$ $b_n = \frac{n \ln n}{n^3} = \frac{\ln n}{n^2}$


$\int_2^{\infty} \frac{\ln x}{x^2} dx = \lim_{b \rightarrow \infty} \left. \frac{-\ln x}{x} \right|_2^b + \int_2^b \frac{1}{x^2} dx$

$$\begin{aligned}
 u = \ln x \quad dv = \frac{1}{x^2} dx & \Rightarrow du = \frac{1}{x} dx \quad v = -\frac{1}{x} \\
 \int u dv = uv - \int v du & = \lim_{b \rightarrow \infty} \left(\frac{-\ln b}{b} + \frac{\ln 2}{2} + \frac{-1}{b} + \frac{1}{2} \right) \\
 & = \frac{1+\ln 2}{2} \text{ since } \lim_{b \rightarrow \infty} \frac{-\ln b}{b} = \lim_{b \rightarrow \infty} \frac{-\frac{1}{b}}{1} = 0
 \end{aligned}$$

$\Rightarrow \sum_{n=1}^{\infty} a_n$ converges and $\lim_{b \rightarrow \infty} \frac{-1}{b} = 0$
 by the Integral Test

$\lim_{n \rightarrow \infty} \left| \frac{a_n}{b_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{n \ln n}{(n-1)^3}}{\frac{\ln n}{n^2}} \right|$
 $= \lim_{n \rightarrow \infty} \left| \frac{n \ln n}{(n-1)^3} \cdot \frac{n^2}{\ln n} \right| = \lim_{n \rightarrow \infty} \left| \frac{n^3}{(n-1)^3} \right| = 1$
 $\sum_{n=1}^{\infty} b_n$ converges so $\sum_{n=1}^{\infty} a_n$
 converges by the Limit Comp. Test.

10.2


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 10.2-10.6 Strategies to deal
 with all tests

Test for Divergence (n^{th} term test):

If $\lim_{n \rightarrow \infty} a_n \neq 0$ or $\lim_{n \rightarrow \infty} a_n$ does not exist, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

geometric series $\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=1}^{\infty} ar^n$ is convergent for $|r| < 1$ and
 divergent if $|r| \geq 1$.

10.3 The Integral Test

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with all tests

If $f(x)$ is: a) continuous, on the interval $[k, \infty)$
b) positive, constant $k > 0$
c) and decreasing

, then the series $\sum_{n=k}^{\infty} a_n$ (with $a_n = f(n)$)

i) is convergent when $\int_k^{\infty} f(x) dx$ is convergent.

ii) is divergent when $\int_k^{\infty} f(x) dx$ is divergent.

p -series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent for $p > 1$ and divergent if $p \leq 1$.

10.4

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with all tests

The Comparison Test:

Given the series $\sum_{n=1}^{\infty} a_n$, ($a_n \geq 0$)

(i) if the terms a_n are **smaller** than the terms b_n of a known **convergent**

series $\sum_{n=1}^{\infty} b_n$ ($b_n \geq 0$), then our series $\sum_{n=1}^{\infty} a_n$ is also **convergent**.

(ii) if the terms a_n are **larger** than the terms b_n of a known **divergent**

series $\sum_{n=1}^{\infty} b_n$ ($b_n \geq 0$), then our series $\sum_{n=1}^{\infty} a_n$ is also **divergent**.

10.4

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The Limit Comparison Test:

Given the series $\sum_{n=1}^{\infty} a_n$, ($a_n > 0$) and a

known convergent or divergent series $\sum_{n=1}^{\infty} b_n$, ($b_n > 0$)

If the $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c$ where c is a finite positive number, then

the series will behave alike, i.e. either both converge or both diverge.

10.5 The Ratio Test

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with all tests

Let $\{a_n\}$ be a sequence and assume that the following limit exists: $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$

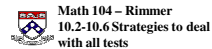
i) If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

ii) If $L > 1$ or if the limit is infinite, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

iii) If $L = 1$, the Ratio Test is inconclusive.

(the series could be absolutely convergent, conditionally convergent, or divergent)

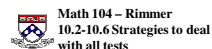
10.5 The Root Test



Let $\{a_n\}$ be a sequence and assume that the following limit exists: $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L$

- i) If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.
- ii) If $L > 1$ or if the limit is infinite, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.
- iii) If $L = 1$, the Root Test is inconclusive.
(the series could be absolutely convergent, conditionally convergent, or divergent)

10.6 The Alternating Series Test



If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ (where $b_n > 0$) satisfies:

- i) $\lim_{n \rightarrow \infty} b_n = 0$
 - ii) $\{b_n\}$ is a decreasing sequence, and
- , then the series is **convergent**.