MATH 103 - MIDTERM I

You must show all work, even on multiple choice questions, or no credit will be given.

(1) Evaluate the limit.
$$\lim_{x \to -4} \frac{2x^2 + 5x - 12}{5x + 20}$$

   (a) 0
   (b) 1
   (c) 1/2
   (d) $-11/5$
   (e) -11
   (f) $-7/5$
   (g) -7
   (h) Does not exist.

(2) Evaluate the limit.
$$\lim_{x \to 0} \frac{\sqrt{x^2 + 9} - 3}{x^2}$$

   (a) 0
   (b) 2
   (c) 1/2
   (d) 1/3
   (e) $-1/3$
   (f) 1/6
   (g) $-1/2$
   (h) Does not exist.

(3) Using the definition of derivative, find the derivative of
$$f(x) = \frac{1}{\sqrt{x}}$$

(4) True or false and explain your answer: There is at least one point x between 0 and $\pi/2$ where $\cos(x) + \sin(x) = x^2$.

(5) True or false and explain your answer: There is at least one point $x \in [0, \infty)$ where
$$e^x - \frac{1}{2(x^2 + 1)} = 0$$

(6) Find the values of a and b that make the function $f(x)$ continuous.

$$f(x) = \begin{cases} \sin(x)/x & x < 0 \\ ax + b & 0 \le x \le 2 \\ x^2 + 3 & x > 2 \end{cases}$$

(7) Using the rules of differentiation, match the function with its derivative.

Function

a) $5x^3 - 2x^2 + 12$ b) $\frac{x^2+1}{\cos(x)}$

c) $\frac{x^2-1}{x+1}$ d) $\frac{1}{(x-2)(x-3)}$

Derivative

i) $15x^3 - 4x^2$ ii) $\frac{x^2+2x-1}{(x+1)^2}$

iii) 1 iv) $((x^2 + 1)\tan(x) + 2x)\sec(x)$

v) $15x^2 - 4x$ vi) $\frac{1}{(x-3)^2(x-2)^2}$

vii) $\frac{5-2x}{(x-2)^2(x-3)^2}$ viii) $\frac{6}{(x-2)^2(x-3)^2}$

ix) Not found   x) $\frac{2x\cos(x)-x^2\cos(x)}{\cos^2(x)}$

xi) $\frac{2x}{(x+1)^2}$ xii) $\frac{2x}{\cos^2(x)}$

(8) Suppose $g(x)$, $h(x)$, and $j(x)$ are differentiable function with the values of the function and its derivative listed in the following table.

x	$g(x)$	$h(x)$	$j(x)$	$g'(x)$	$h'(x)$	$j'(x)$
-1	3	0	1	-1	-2	-2
0	2	3	0	-2	3	-2
1	0	-1	-2	-2	-2	-1
2	-2	-2	-3	-1	0	2
3	-3	0	1	0	1	2

Use the table to calculate the following values:

(a) If $f(x) = 3j(x) - 2h(x)$, find $f'(3)$.

(b) If $f(x) = g(x) \cdot h(x)$, find $f'(0)$.

(c) If $f(x) = g(x)(h(x) + j(x))$, find $f'(2)$.

(d) If $f(x) = h(x)j(x)g(x)$, find $f'(1)$.

(e) If

$$f(x) = \frac{3h(x)}{g(x) + j(x)}$$

find $f'(0)$.

(9) What is the best linear approximation to $f(x) = \sqrt{x}$ at $x = 4$. Use this to approximate $\sqrt{4.1}$.