The Wave Equation in $\mathbb{R}^2$ and $\mathbb{R}^3$

To Solve

$$u_{tt} = c^2 \Delta u, \qquad x \in \mathbb{R}^n$$

with

$$u(x,0) = \phi(x), \quad u_t(x,0) = \psi(x)$$

In $\mathbb{R}^3$ [Poisson: *Kirchoff's formula*]

$$u(x_0,t_0) = \frac{1}{4\pi c^2 t_0} \iint_S \psi(x)\, dS + \frac{\partial}{\partial t_0}\left[\frac{1}{4\pi c^2 t_0} \iint_S \phi(x)\, dS\right],$$

where S is the sphere centered at x_0 with radius ct_0.

In $\mathbb{R}^2$. Here we write $(x,y) \in \mathbb{R}^2$.

$$u(x_0,y_0,t_0) = \frac{1}{2\pi c} \iint_D \frac{\psi(x,y)}{[c^2t_0^2 - (x-x_0)^2 - (y-y_0)^2]^{1/2}}\, dx\, dy$$
$$+ \frac{\partial}{\partial t_0}[\text{same expression with } \phi],$$

where D is the disk $\quad (x-x_0)^2 + (y-y_0)^2 \le c^2 t_0^2$.