Math 425/525, Spring 2011 **Jerry L. Kazdan**

Problem Set 2

DUE: In class Thursday, Feb. 3 *Late papers will be accepted until 1:00 PM Friday.*

1. Find the solution $U(t) := (u_1(t), u_2(t))$ of

$$\begin{aligned} u_1' &= u_1 \\ u_2' &= u_1 - u_2 \end{aligned}$$

with the initial conditions $U(0) = (u_1(0), u_2(0)) = (1\,0)$.

2. Find the solution $U(t)$ of

$$\begin{aligned} u_1' &= 2u_1 - u_2 \\ u_2' &= 3u_1 - 2u_2 \end{aligned}$$

with $U(0) = (0, 1)$.

3. By only a slight modification of your solution of the previous problem, find the general solution of

$$\begin{aligned} u_1'' &= 2u_1 - u_2 \\ u_2'' &= 3u_1 - 2u_2 \end{aligned}$$

4. One simple model of a diatomic molecule is of masses m and M connected by a spring with spring constant k so the equations of motion are

$$\begin{aligned} mu_1'' &= k(u_2 - u_1) \\ Mu_2'' &= k(u_1 - u_2) \end{aligned}$$

Solve these equations by finding the normal modes of oscillation. Interpret your results with a diagram.

5. a) Find the solution $u(x,y)$ of $u_x + 3u_y - u = 0$ with $u(0,y) = 1 + y^2$.
   b) Find the solution $v(x,y)$ of $v_x + 3v_y - v = 1$ with $v(0,y) = 1 + y^2$.

[Last revised: January 29, 2011]